Errata to Diffusion Under Confinement: A Journey Through Counterintuition

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1 Chapter 3

1. Page 74. In Eq. (3.134) the arguments of ρ should be ξ and τ . Consequently, the equation should read $\rho(\xi, \tau)$.

2 Chapter 4

1. Page 89. In the first line after Eq. (4.30) the arguments of p should be x and t = 0. Consequently, the equation should read p(x, t = 0).

3 Chapter 16

1. Page 494. The second line after Eq. (15.64) should read:

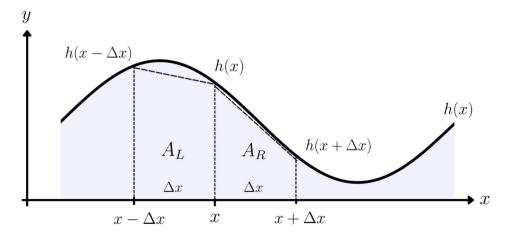
the cylinder wall, πR^2 , consequently, $\kappa = 4Da/(\pi R^2)$..

2. Page 494. The first line after Eq. (16.5) should read:

where $\sigma = \pi a^2/(\pi R^2)$, and the normalized variable ν is given by $\nu = a/R = \sqrt{\sigma}$,

4 Chapter 17

1. Page 516. Figure 17.1 should be as follows:



2. Page 517. In Eq. (17.6) the second term of the right-hand side should be h not A, consequently, it should read:

$$A_R = \frac{\Delta x}{2} \left[h(x) + h(x + \Delta x) \right]$$

3. Page 518. In Eq. (17.7) the second term of the right-hand side should be h not A, consequently, it should read:

$$A_L = \frac{\Delta x}{2} \left[h(x) + A(x - \Delta x) \right],$$

4. Page 518. In Eq. (17.8) the argument of h should be independent on time, consequently, it should read:

$$A_R = \frac{\Delta x}{2} \left[2h(x) + \Delta x \frac{\partial h(x)}{\partial x} + \frac{(\Delta x)^2}{2} \frac{\partial^2 h(x)}{\partial x^2} + \cdots \right], \tag{1}$$

5. Page 518. In Eq. (17.9) the argument of h should be independent on time, consequently, it should read:

$$A_L = \frac{\Delta x}{2} \left[2h(x) - \Delta x \frac{\partial h(x)}{\partial x} + \frac{(\Delta x)^2}{2} \frac{\partial^2 h(x)}{\partial x^2} + \cdots \right],$$
(2)

6. Page 518. In Eq. (17.14) the left-hand side should read:

$$b(x) = \frac{A_L}{A_T}$$

7. Page 520. In Eq. (17.24) the right-hand side the area in the denominator of the partial derivative should be $\mathcal{A}(x)$, not A(x), consequently, it should read:

$$J(x,t) = -D_0 \mathcal{A}(x) \frac{\partial}{\partial x} \frac{\rho(x,t)}{\mathcal{A}(x)}.$$

8. Page 521. In Eq. (17.25) a partial derivative respect to x is missing, the last term of the right-hand side should read:

$$\frac{\mathcal{A}'(x)}{\mathcal{A}(x)}\frac{\partial\rho(x,t)}{\partial x}$$

9. Page 521. For the sake of simplicity, after Eq. (17.24) should read:

By substituting Eq. (17.24) into the continuity equation, we find that

$$\begin{aligned} \frac{\partial \rho(x,t)}{\partial t} &= \frac{\partial}{\partial x} \bigg\{ D_0 \mathcal{A}(x) \frac{\partial}{\partial x} \bigg[\frac{\rho(x,t)}{\mathcal{A}(x)} \bigg] \bigg\} \\ &= D_0 \bigg\{ \frac{\partial^2}{\partial x^2} \bigg[\frac{\rho(x,t)}{\mathcal{A}(x)} \bigg] + \mathcal{A}'(x) \frac{\partial}{\partial x} \bigg[\frac{\rho(x,t)}{\mathcal{A}(x)} \bigg] \bigg\}, \end{aligned}$$

which yields to

$$\frac{\partial P(x,t)}{\partial t} = D_0 \bigg[\frac{\partial^2 P(x,t)}{\partial x^2} + \frac{\mathcal{A}'(x)}{\mathcal{A}(x)} \frac{\partial P(x,t)}{\partial x} \bigg],$$

with $P(x,t) \equiv \rho(x,t)/\mathcal{A}(x)$.

10. Page 521. Equations (17.25)-(17.29) should change $\rho(x,t)$ for P(x,t). Thus, these equations should read:

$$\frac{\partial P(x,t)}{\partial t} = D_0 \left[\frac{\partial^2 P(x,t)}{\partial x^2} + \frac{\mathcal{A}'(x)}{\mathcal{A}(x)} \frac{\partial P(x,t)}{\partial x} \right].$$
(17.25)

$$\frac{\partial^2 P(x)}{\partial x^2} + \frac{2}{x} P(x) = 0.$$
 (17.26)

$$P(x) = C_2 - \frac{C_1}{x}.$$
(17.27)

Evaluating the constants in this equation from the boundary conditions, namely, $P(x = h) = P_0$ and P(x = h + H) = 0, the solution becomes

$$P(x) = \frac{h}{H} \left(\frac{h+H}{x} - 1 \right) P_0.$$
 (17.28)

These results show that density is proportional to P_0 and inversely proportional to the length of the funnel H and does not depend on diffusivity because of the steady state.

:
$$J = \frac{D_0 \pi m^2 h (h+H)}{H} P_0.$$
(17.29)

5 Chapter 18

1. Page 544. The title of Section 18.5.2 should read: "Mean Square Displacement and Transient Behavior".

6 Appendix A.8

1. Page 733. In Eq. (A.56), the right-hand side should read $\mathcal{L}{f(t)}$ instead of $\Lambda{f(t)}$.