Elliptical Chemoreceptors: The key to an Effective Absorption

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Abstract

Models of molecules transport through diffusion offer an important insight into the description of microscopic phenomena in nature. In the research, we focused on some models of chemoreception, from the perfect spherical absorbent, the use of Weber's disk, and the development of Berg and Purcell approach, to Zwanzig and Szabo generalized solution where interference effect and partially absorbing receptors are considered. Afterwards, we extrapolate Dudko's solution using a dimensional comparison for rate constant diffusion to receptors of arbitrary shape on a spherical cell, and contrast the absorption effectiveness on circular and elliptical chemoreceptors, using as reference the perfect spherical absorbent. During this process we looked to an important property related with the structure of chemoreceptors, their geometry. Is there a preferential setting that allows more particles to be absorbed in an ever smaller area? We found that the elliptical geometry offers a plausible model in cellular anatomy, a result that could explain the structure variation on chemoreceptors and the observed physiological changes on cells.

Keywords: Cells, membranes, diffusion, biological models, biophysics, chemoreceptors.

1. Introduction

It is well known that cells are a fundamental part in the existence of life, a little organic machine capable of making big things, thanks to which we can breathe, move and reproduce. Not mentioning, that cells are responsible of the embryogenesis process. In addition to the existentiality of cells it is necessary that they work correctly, and most of the times, that depends on the cellular intercommunication, a mechanism that can be reduced into three steps; the reception, transduction and response. By the scope of the research we concentrate on the first step, the reception of signaling molecules or ligands. These particles carry useful information to the cells causing them to migrate, reproduce, or even die. A schematic representation of the cellular intercommunication is shown in Fig. 1.

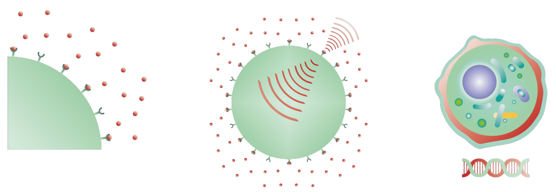

Figure 1: Schematic representation of the intercellular information exchange sequence. From left to right, receptor-ligand interaction, transduction (amplification of molecular signals) and specific physiological response, DNA transcription in the cell nucleus are represented.

Guided by chemical reactions, cells are a complex system that create and dismantle structures, besides making perfect copies of themselves. This biochemical network could go wrong when tiny mistakes add up until the machinery gets corrupted, something that could lead into biological disorders, like cancer. Intercellular communication is as important as the existence of cells, and the interaction receptor-ligand is the very first part of this process, a phenomenon that can be modeled with diffusion equations.

2. A counter-intuitive mental experiment

Before diving into the research details we set a mental experiment related with the absorption of particles. Suppose we have two spheres, one of which has one reflective and one absorbent hemisphere, while the second has tiny, uniformly distributed absorbent circular patches on the surface, so that together, occupy a minimal area on the cell membrane, say 6 thousandths of it. The question is: Which configuration can absorb more molecules if they are immersed in a fluid with an specific concentration gradient?

The answer seems to be obvious, as the first sphere has a much larger absorbent total area than the second one. In 1977, Berg and Purcell gave a counter-intuitive solution to this chemoreception problem [1], The two configurations can absorb exactly the same!

3. Evolution of a model

The chemoreception model has been modified through the years [3,4,5], in order to approximate it to reality as much as possible, like the Zwanzig modification, in the interference effect [4], and the introduction of patches with selectivity information by Zwanzig and Szabo [5]. The latter being

$$\frac{1}{k_{ZS}} = \frac{1}{k_S} + \frac{(1-\sigma)}{Nk_H} + \frac{1}{Nk\pi s^2} \qquad (1)$$

A result derived from the use of the Smoluchowski constant,

$k_S = 4\pi Da$, associated with the perfect spherical absorbent, and the Hill constant, $k_H = 4Ds$, which comes from the Weber's disk; where a and s are the radii of the sphere and of the receptors, respectively and D is the diffusion coefficient, which is considered to be constant. On the other hand, σ is the occupied area by the patches, N the number of receptors and k the permeability of the system. It's important to mention that we are interested in the constant current diffusion because it will tell us the number of particles absorbed per unit time, a perfect parameter to measure the nutrition of the cell. Circular patches were used in all of the modifications, but, does the geometry of the receptors have a considerable effect on the absorption capacity? A theoretical formulation of non-circular receptors is necessary.

4. The importance of geometry

During the investigation, the diffusion current constant associated with an arbitrarily shaped patch on a reflecting plane, k_G, was calculated, through a comparison between the analytically calculated constant for an elliptical receptor in terms of the eccentricity ϵ and the semi-major axis a_1

$$k_\epsilon = \frac{2\pi D a_1}{k_\epsilon(\epsilon)}, \qquad k_\epsilon(\epsilon) = \int \frac{d\theta}{\sqrt{1-\epsilon^2 \sin^2\theta}} \tag{2}$$

in contrast to a purely geometric formulation, which is correctly reduced to circular patches when the area A and perimeter P are given, namely

$$k_{GP} = \frac{2^{1+2\nu}}{\pi^{1-\nu}} A^\nu P^{1-2} \tag{3}$$

A quantity that depends on a numerical parameter ν. By doing this comparison it is found that

$$k_G = \left(\frac{2^5 AP}{\pi^2}\right)^{1/3} D \tag{4}$$

a result that matches with what is reported by Dudko Et al. [2]. After extrapolate the results, the constant current diffusion associated to N patches of area A_R on a spherical surface is

$$\frac{1}{k_{AS}} = \frac{1}{k_S} + \frac{(1-\sigma)}{Nk_G} + \frac{1}{NkA_R} \tag{5}$$

When the interference effect is neglected, the Eqn (5) becomes

$$\frac{1}{k_{PD}} = \frac{1}{k_S} + \frac{1}{Nkk_G} \tag{6}$$

Circular, square, triangular, hexagonal, n-sided polygons, and elliptical patches were treated, all on a reflecting spherical surface. Each showed less effectiveness compared to the circular patches, except the elliptical ones. When using receptors with a permeability of 50%, the number of patches needed to reach the absorbing hemisphere is $N_c = 31,416$, for circular receptors and $N_\epsilon = 31,337$, for elliptical receptors. Where we set the semi-minor axis equals the radius of the circle and $\epsilon = 0.1$. In this way, elliptical receivers take up only 0.25% more space relative to circular ones.

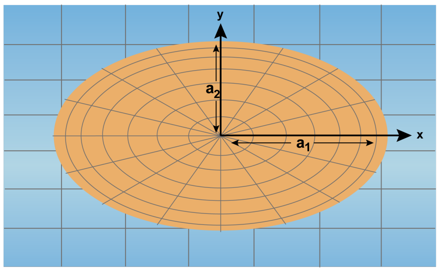

Figure 2: Elliptical chemoreceptor with semi-major axis a_1, semi-minor axis a_2 and eccentricity ϵ on a reflective surface.

The space-absorption ratio is good, but this can be attributed to the fact that an ellipse with this configuration actually has more area than a circle and would ultimately take up more space on the membrane. Nevertheless, when fixing the individual area of the receptors, that is, the area of the circle equal to that of the ellipse and making a relationship between the current constants, the following expression is found

$$\frac{k_{G\epsilon}}{k_{Gc}} = \left[\frac{1-\frac{\epsilon^2}{4}-\frac{3}{64}\epsilon^4}{(1-\epsilon^2)^{1/4}}\right]^{1/3} \tag{7}$$

where $k_{G\epsilon}$ is that of elliptical receptors and k_{Gc} of the circular ones. This number is always greater than one, $k_{G\epsilon} > k_{Gc}$ $\forall$ $\epsilon > 0$, the absorption capacity is greater if an elliptical geometry is considered, they occupy the same amount of area on the plane but can absorb more signaling molecules.

5. Conclusions

A chemoreceptor distribution is as good as a perfect absorbent hemisphere and it can be better if elliptical receptors are considered. Furthermore, the most effective geometry will be the one with the largest perimeter once the area has been set. It is necessary to recognize the importance of geometry in cellular intercommunication, which could have great applications in bioengineering. Meanwhile, this result is left as an irrefutable improvement to the Berg and Purcell model. Elliptical membrane receptors, the key to effective absorption.

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