DECEMBER 1-3, 2021.
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IWoSP: INTERNATIONAL WORKSHOP ON STATISTICAL PHYSICS

Creating spaces to highlight and discuss about statistical physics' research

Connection between random networks and nonextensive statistical mechanics

Constantino Tsallis[1,2,3*]

[1] *Centro Brasileiro de Pesquisas Fisicas and INCT-SC, Rua Xavier Sigaud 150, Rio de Janeiro 22290-180, Brazil*
[2] *Santa Fe Institute, 1399 Hyde Park Road, Santa Fe, New Mexico 87501, USA*
[3] *Complexity Science Hub Vienna, Josefstädter Strasse 39, 1080 Vienna, Austria*
[*] tsallis@cbpf.br

Boltzmann-Gibbs statistical mechanics, based on the additive BG entropy, exhibits various isomorphisms with random systems, e.g., the Kasteleyn-Fortuin theorem for bond percolation, the de Gennes isomorphism for self-avoiding random walk, equivalence with random networks of impedances. Similarly, nonextensive statistical mechanics, based on the nonadditive entropy S_q, is connected (see [1-3] and references therein) with the so-called (asymptotically) scale-free random networks. We present here a computational study [3] of one such d-dimensional geographic model with a preferential-attachment probability decaying like $1/distance^{\,\alpha_A}$ ($\alpha_A \geq 0$) with links weighted through the distribution $P(w) \propto e^{-(w/w_0)^{\eta}}$ ($w_0 > 0$, $\eta > 0$). We exhibit that the distributions of energy ϵ are very satisfactorily fitted by q-exponentials $e_q^{-\beta_q \epsilon} \equiv 1/[1 + (q-1)\beta_q \epsilon]^{1/(q-1)}$ ($q \geq 1$, $\beta_q > 0$), which optimize the entropy S_q under simple constraints. In Fig. 1 we illustrate the model, the quality of the fittings, and the α/d-dependence of (q, β_q). Notice that the dependences on (α_A, d) only act through the ratio α_A/d, and that the entropic index q does not depend on (η, w_0), thus characterizing universality classes.

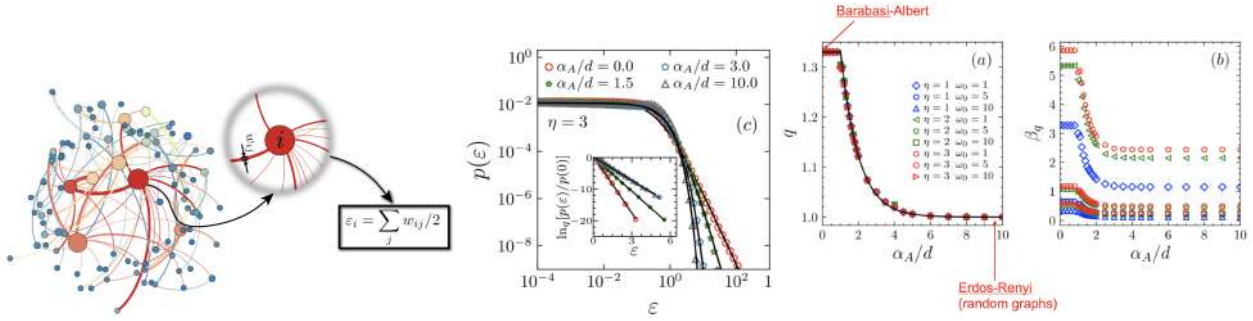

Figura 1: *Left: Model. Center: Energy distributions fitted by q-exponentials. Right:* (q, β_q) *parameters as functions of* (α/d).

Acknowledgment

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II International Workshop on Statistical Physics
Antofagasta, Chile, December 1-3, 2021 Topic(s): **Complex systems**

Characterization of Visuomotor/Imaginary Movements in EEG: An Information Theory and Complex Network Approach

Roman Baravalle[1], Natalí Guisande[1], Mauro Granado[1] Osvaldo A. Rosso[2*] and Fernando Montani[1]

[1] *Instituto de Física La Plata (IFLP), CONICET CCT-La Plata and Universidad Nacional de La Plata (UNLP), La Plata, Argentina*

[2] *Instituto de Física, Universidade Federal de Alagoas, Maceió, Brazil.*

[*] oarosso@gmail.com

Imagined activities could actually be a cognitive basis for creative thinking. However, it is still unknown how they might be related with the architecture of the brain. A recent study has proved the relevance of the imagined activity when investigating neuronal diseases by comparing variations in the neuronal activity of patients with brain diseases and healthy subjects. One important aspect of the scientific methodologies focused on neuronal diseases is therefore to provide a trustablemethodology that could allow us to distinguish between realized and imagined activities in the brain. The electroencephalogram is the result of synchronized action of the cerebrum, and our end is portraying the network dynamics through the neuronal responses when the subjects perform visuomotor and specific imaginary assignments. We use a subtle information theoretical approach accounting for the time causality of the signal and the closeness centrality of the different nodes. More specifically we perform estimations of the probability distribution of the data associated to each node using the Bandt and Pompe approach to account for the causality of the electroencephalographic signals. We calculate the Jensen-Shannon distance across different nodes, and then we quantify how fast the information flow would be through a given node to other nodes computing the closeness centrality.We performa statistical analysis to compare the closeness centrality considering the different rhythmic oscillation bands for each node taking into account imagined and visuomotor tasks. Our discoveries stress the pertinence of the alpha band while performing and distinguishing the specific imaginary or visuomotor assignments.

Climate Meets Complex Systems: from 2021 Physics Nobel Price Winners to Recent Directions

Juergen Kurths

Potsdam Institute for Climate Impact Research & Humboldt University, Department of Physics, Berlin, Germany

Juergen.Kurths@pik-potsdam.de

The Earth system is a very complex and dynamical one basing on various feedbacks. This makes predictions and risk analysis even of very strong (sometime extreme) events as floods, landslides, heatwaves, earthquakes etc. a challenging task. After introducing physical models for weather forecast already in 1922 by Richardson, a strong problem has been the understanding of basic physical mechanisms and exploring anthropogenic influences on climate. In 2021 Hasselmann and Manabe got the Physics Nobel Price for their pioneering works on this. I will shortly review their main contributions. Next, I will introduce a recently developed approach via complex networks mainly to analyze strong climate events. This leads to an inverse problem: Is there a backbone-like structure underlying the climate system? For this we propose a method to reconstruct and analyze a complex network from observational and reanalysis data. This approach enables us to uncover relations to global and regional circulation patterns in oceans and atmosphere, which leads to construct substantially better predictions, in particular of the onset of the Indian Summer Monsoon and El Niño.

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II International Workshop on Statistical Physics
Antofagasta, Chile, December $1^{st} - 3^{rd}$, 2021 Topic(s): (**Statistical Physics and fundamentals.**)

Statistical Physics, Complexity, and Us

Víctor Muñoz[1*]
[1] *Departamento de Física, Facultad de Ciencias, Universidad de Chile*
[*] vmunoz@fisica.ciencias.uchile.cl

Introduction

Physics intends to understand our environment. The classical approach, the approach which is usually taught at school, and the one which most people may be familiar with, is to model physical systems by means of some equations and, given certain initial conditions, find the trajectory. Cars moving in a highway, balls falling from a given height, or masses attached to springs, are typical problems that may be solved this way, and may even represent the image of Physics many people has by the end of high-school years.

But, what if the system has so many particles that finding individual trajectories is impractical or even useless? What if correlations between particles turn the problem into an unsolvable set of equations? And if initial conditions are not exactly known, and small changes in them yield completely different results?

These issues lead us to the rich universe of statistical physics, nonlinear physics, chaos, and complexity. Concepts that have enlarged the fields of interest of Physics, pushing it into the realm of interdisciplinary research. In this talk, we will explore some of the consequences of this, and what Physics has to tell us about rabbits, coast lines, weather, and coffee. What Statistical Physics has to do with us.

Acknowledgment

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Is There a Thermodynamics of Nonequilibrium Steady States?

Ronald Dickman[1*]

[1] *Universidade Federal de Minas Geras and National Institute of Science and Technology for Complex Systems*
[*] dickman@fisica.ufmg.br

Introduction

Over the past 20 years, researchers have tried to construct a macroscopic theory of nonequilibrium steady states (NESS) analogous to equilibrium thermodynamics. I shall describe an approach based on the definition of intensive parameters such as temperature and chemical potential for NESS via coexistence with reservoirs [1]. This program has enjoyed some success in predicting the properties of initially isolated systems when brought into contact. Nevertheless, consistent treatment of nonuniform systems, including coexisting phases, appears to be beyond the scope of the theory [2,3]. This talk will review these issues, as well as questions about a nonequilibrium entropy function, and reservoir equivalence, in the context of stochastic lattice models [4]. Our results suggest that the thermodynamics of NESS is far more limited than that of equilibrium systems.

Acknowledgment

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Reaction diffusion fronts with a free boundary: the speed of the advancing front

R. D. Benguria[1,†], M. C. Depassier [1*]
[1] *Instituto de Física, Pontificia Universidad Católica de Chile*
†rbenguri@uc.cl *mcdepass@uc.cl,

Summary

The propagation of reaction diffusion fronts in a system with a free boundary is a topic of current interest since it constitutes a more accurate model of physical and biological problems where a clear position for the advancing front, be it a flame, population or temperature, can be observed. In contrast, in the classical Fisher type reaction diffusion problems the front extends to infinity for all $t > 0$. In this work we study a moving boundary with Stefan boundary conditions and use a variational approach to determine the speed of the advancing front. Our results are valid for general monostable and combustion reaction terms.

The problem

We study the reaction diffusion equation in one dimension with Stefan boundary conditions,

$$\begin{aligned} &u_t = u_{xx} + f(u) \text{ with } f(0) = f(1) = 0, \\ &u_x(0,t) = 0, \quad u(L(t),t) = 0, \quad \frac{dL(t)}{dt} = -\kappa u_x(L(t),t), \end{aligned} \tag{1}$$

where $L(t)$ is the free boundary and subscripts denote derivatives with respect to the independent variables x and t. The reaction term is either monostable or of combustion type. It has been shown that depending on the initial conditions $L(t=0)$ and $u(x,0)$ the perturbation may spread and gradually invade the space or may die away. In the spreading regime there is a unique traveling wave solution $u(x,t) = q(x-ct)$ [1,2]. The problem we address here is the determination of the speed of a spreading front as a function of the reaction term and of the Stefan parameter κ. Spreading traveling wave solutions $u(x,t) = q(x-ct)$ correspond to solutions of the ordinary differential equation

$$q_{zz} + cq_z + f(q) = 0, \text{ with } q(-\infty) = 1,\ q(0) = 0,\ q_z(0) = -c/\kappa$$

where $z = x - ct$.

We show that the speed c of the front can be obtained by means of a variational principle. In addition to provide a method to estimate the speed as accurately as desired, the variational principle allows to study the dependence of the speed on the parameters of the problem and to establish upper and lower bounds for the speed of general validity. For example, we show that

$$\frac{\kappa}{\kappa+1} c_{ZFK} \le c \le \frac{\kappa}{\kappa+1} c_0 \qquad \text{where} \qquad c_{ZFK} = \sqrt{2\int_0^1 f(u)du}$$

is the Zeldovich Frank-Kamenetskii speed and c_0 is the speed of the front in the classical case. It follows from this inequality that $\lim_{\kappa\to 0} c = 0$ and $\lim_{\kappa\to\infty} c = c_0$. Recent work [3] has considered the case of density dependent reaction, our results can be extended to this problem as well.

Acknowledgment

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II International Workshop on Statistical Physics
Antofagasta, Chile, December 01-03, 2021 Topic(s): **Statistical Mechanics and Fundamentals**

Jaynes' "Caliber" is proportional to QGT's "Exertion"

M.C. Parker[1] and C. Jeynes[2]
[1] *University of Essex, Colchester, CO4 3SQ, UK.*
[2] *University of Surrey, Guildford, GU2 7XH, UK*
[1]mcpark@essex.ac.uk, [2]c.jeynes@surrey.ac.uk

Abstract

Quantitative Geometrical Thermodynamics (QGT) has recently emerged as an important new entropic organising principle able to describe and explain many natural phenomena [1,2]. In particular, QGT provides a quantitative analytical framework for the role of entropy in dynamically shaping many natural structures and processes, from spiral galaxies at cosmic scales down to sub-atomic nuclei at the microscopic scale [3]. We have previously shown [1] that certain holomorphic geometries can be represented by entropic Lagrangian/Hamiltonian formulations that correctly obey the Cauchy-Riemann relations in hyperbolic Minkowski spacetime, and which underpin the Principle of Least Exertion (PLE) described by a variational calculus using the Euler-Lagrange equations. The PLE is isomorphic and complementary to the kinematic Principle of Least Action, where "exertion" is the path integral of the entropic Lagrangian that is quantized by the Boltzmann constant k_B; that is to say, exertion is isomorphic to "action", which is the path integral of the kinematic Lagrangian based on the Planck constant $\hbar$.

Jaynes proposed his concept of Maximum Caliber (MaxCal) in 1980 [4,5], based on variational and maximum entropy (MaxEnt) principles. Unfortunately, Jaynes was only able to provide an outline sketch for his theory, since he considered the Maximum Caliber principle to be: "... probably beyond our mathematical ability to do the indicated calculations explicitly for any really nontrivial problem; that is perhaps a task for the computers of the next Century." However, it has become clear that there is a straightforward analytic demonstration in QGT of MaxCal's validity. Recent years has seen a resurgence of interest in Jaynes' MaxCal ideas, with a recent overview by Pressé et al. [6] and application of Maximum Caliber theory to systems near and far from equilibrium [7,8]. Interestingly, Pressé et al. [6] also suggest that the MaxCal quantity should be relativistically invariant. Clearly, such a requirement differs from conventional entropic definitions, but accords with our recent coordinate invariant identification of the entropy production $\dot{S} = \partial S/\partial t$ as the product of the entropic Hamiltonian p_0 and the speed of light, such that $\dot{S} = cp_0$ [9]. We now demonstrate that the varying part of the caliber C is proportional to what we have previously defined as the entropic exertion X, with the coefficient of proportionality being the entropic mass m_S : $C = -X/m_S$. Since the exertion is simply the path integral of the entropic Lagrangian L_S, with its variation given by $\delta X = \delta(\int L_S dx) = 0$ is clear that Jaynes was correct when he wrote [5]: *"The caliber of a space-time process thus appears as the fundamental quantity that "presides over" the theory of irreversible processes in much the same way that the Lagrangian presides over mechanics."*

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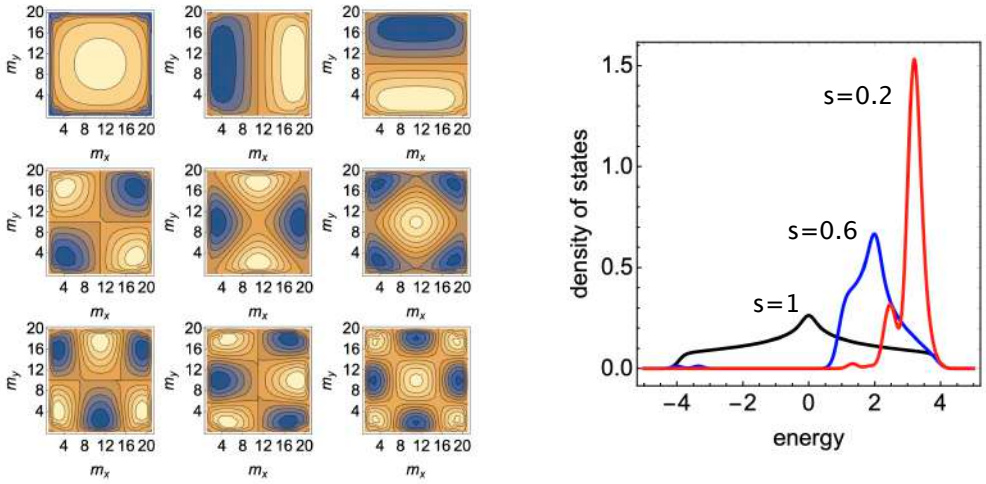
II International Workshop on Statistical Physics
Antofagasta, Chile, December 1-3, 2021 Topic(s): **Statistical mechanics and fundamentals**

The fractional discrete nonlinear Schrödinger equation

Mario I. Molina
Departamento de Fisica, Facultad de Ciencias, Universidad de Chile
*mmolina@uchile.cl

Introduction

We study a fractional version of the $1D$ and $2D$ discrete nonlinear Schrodinger (DNLS) equation, where the usual discrete Laplacian is replaced by its fractional form that depends on a fractional exponent s that interpolates between the case of an identity operator ($s = 0$) and that of the usual discrete 2D Laplacian ($s = 1$). This replacement leads to a long-range coupling among sites that, at low values of s, decreases the bandwidth and leads to quasi-degenerate states. The mean square displacement of an initially-localized excitation is shown to be ballistic at all times with a 'speed' that increases monotonically with the fractional exponent s. We also compute the nonlinear modes and their stability for both, bulk and surface modes. The modulational stability is seen to increase with an increase in the fractional exponent. The trapping of an initially localized excitation shows a selftrapping transition as a function of nonlinearity strength, whose threshold increases with the value of s. In the linear limit, there persists a linear trapping at small s values. This behavior is connected with the decrease of the bandwidth and its associated increase in quasi-degeneracy.



Left: Lowest lying modes for a square lattice with fractionality $s = 0.2$ Right: Normalized density of states for a square lattice with different fractional exponents. ($N = 21 \times 21$)

Acknowledgment

We acknowledge support from Fondecyt grant 1200120.

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Incidence of evaporation on the thermodynamics of astrophysical systems: γ-exponential models

Y. J. Gomez-Leyton[1*], L. Velazquez [1,†].
[1] *Departamento de Física, Universidad Católica del Norte, Av. Angamos* 0610*, Antofagasta, Chile*
[*] ygomez@ucn.cl, [†] lvelazquez@ucn.cl

Introduction

The γ-exponential models is a parametric family of lowered isothermal models that attempt to describe the thermo-statistical properties of a self-gravitating systems that evolves along a quasi-stationary evolution under the incidence of evaporation (escape) of their constituents. A keystone for this extension is the special function,

$$E\left(x;\gamma\right)=H\left(x\right)\sum_{k=0}^{\infty}\frac{x^{\gamma+k}}{\Gamma\left(\gamma+1+k\right)}, \tag{1}$$

which represents a continuous deformation via the real parameter γ of ordinary exponential function [1]. Here, $H\left(x\right)$ and $\Gamma\left(x\right)$ are the Heaviside step and Gamma functions, respectively. Its single-mass distribution function for degenerate non-relativistic particles is given by,

$$f_{\gamma}\left(\mathbf{r},\mathbf{p}|\beta,\alpha,\boldsymbol{\omega},\varepsilon_{c}\right)=\frac{\exp\left[-\beta\left(\alpha\,\mathbf{L}^{2}+\boldsymbol{\omega}\cdot\mathbf{L}\right)\right]}{\eta+\exp\left[x\right]}\,E\left(x;\gamma\right), \tag{2}$$

which comprises a series of realistic features of different models existing in the literature [2-6]. Here, the dimensionless variable $x=\beta\left[\varepsilon_{c}-\varepsilon\left(\mathbf{r},\mathbf{p}\right)\right]$ is defined from the individual mechanical energy of particles $\varepsilon\left(\mathbf{r},\mathbf{p}\right)=\mathbf{p}^{2}/2\,m+m\,\varphi\left(\mathbf{r}\right)$, the inverse temperature-like parameter β, and the threshold energy ε_{c} for the evaporation. The function $\mathbf{L}=\mathbf{r}\times\mathbf{p}$ is the angular momentum, whose presence consider the rotation anisotropy via the parameters $(\alpha,\boldsymbol{\omega})$. The first parameter α controls the radial anisotropy, while the second $\boldsymbol{\omega}$ the axial anisotropy. Finally, the term η is a normalization constant that takes into consideration the quantum degeneration, which can be expressed in terms of the Fermi energy ε_{F} as follows,

$$\eta=\exp\left[\beta\left(\varepsilon_{c}-\varepsilon_{F}\right)\right]. \tag{3}$$

In this work, we shall provide a panoramic view about the thermodynamics of these models considering the special solutions studied in the literature.

Acknowledgments

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Application of Generalized Logarithn and Exponential Functions in Multifractal Detrended Fluctuation Analysis (MFDFA) of Nonstationary Time Series for Medical Signal Analysis

Suzielli Mendonça[1,*], Alexandre Martinez[2,†]

[1] *Faculdade de Filosofia, Ciências e Letras de Ribeirão Preto (FFCLRP), Universidade de São Paulo (USP).*

[*] suzielli@usp.br, [†] asmartinez@usp.br

Introduction

Over the last decades, multifractal analysis, via detrended fluctuation analysis (DFA), have been applied extensively in both segmentation and characterization processes, in medical signals. There is a vast range of its applications, which go from electro-cardiogram (ECG), to bone imaging [1]. Regarding ECG signals, the multifractal analysis of ECG data reported are popularly made by using the method of detrended fluctuation analysis, which can quantify the variability in the scalling of the fluctuations in data [2]. The multifractal detrended fluctuation analysis of non-stationary time series (MFDFA) method generalizes DFA method and, by now, is widely used to avoid spurious detection of correlations arising from process trends [3]. It has been already analysed how variations in ECG signals can be detected through multifractal analysis, in which the multifractal analysis of the ECG signals has helped to distinguish healthy and unhealthy cases [4].

Here, our goal is to pursuit more efficient methods to deal with non-stationary time series. Firstly, we rewrite the complicated formulas of the MFDFA algorithms in terms of the Hölder mean. Next, we show that the Hölder mean can be suitably written in terms of the generalized logarithm and exponential functions of the non-extensive statistical mechanics [5-6]. Thus, we write the MFDFA algorithm in terms of these generalized functions, making it compact and ready for implementation. Finally, we have tested our implementation in ECG signals from the PhysioNet public database to validate the algorithm.

Acknowledgments

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Divergence theorem in Bayesian probability under constraints

Sergio Davis[1,2*], Gonzalo Gutiérrez[3,†]

[1] *Research Center on the Intersection in Plasma Physics, Matter and Complexity, P²MC, Comisión Chilena de Energía Nuclear, Santiago, Chile.*

[2] *Departamento de Física, Facultad de Ciencias Exactas, Universidad Andres Bello, Santiago, Chile.*

[3] *Departamento de Física, Facultad de Ciencias, Universidad de Chile, Santiago, Chile.*

[*] sergio.davis@cchen.cl, [†] gonzalogutierrez@uchile.cl

Introduction

The application of the divergence theorem on continuous probability densities with compact support leads to the *conjugate variables theorem* (CVT) [1, 2], namely the identity

$$\left\langle \frac{\partial}{\partial \boldsymbol{x}} \cdot \boldsymbol{\omega} \right\rangle_I + \left\langle \boldsymbol{\omega} \cdot \frac{\partial}{\partial \boldsymbol{x}} \ln P(\boldsymbol{x}|I) \right\rangle_I = 0 \tag{1}$$

where $\boldsymbol{\omega}(\boldsymbol{x})$ is an arbitrary, differentiable vector field. The CVT can be used to derive the equipartition theorem of classical statistical mechanics, as well as several formulas such as Rugh's temperature [3, 4],

$$\left\langle \nabla \cdot \left[\frac{\boldsymbol{v}}{\boldsymbol{v} \cdot \nabla H} \right] \right\rangle_E = \frac{1}{k_B T(E)}. \tag{2}$$

We can augment this identity by including a set of m constraints of the form $g_j(\boldsymbol{x}) = G_j$ $(j = 1, \ldots, m)$ and considering a Bayesian update from the state of knowledge I to $(\boldsymbol{G}, I)$ by virtue of Bayes' theorem in the form

$$P(\boldsymbol{x}|\boldsymbol{G}, I) = \frac{P(\boldsymbol{x}|I) P(\boldsymbol{G}|\boldsymbol{x}, I)}{P(\boldsymbol{G}|I)}, \tag{3}$$

obtaining then the versatile identity

$$\left\langle \frac{\partial}{\partial \boldsymbol{x}} \cdot \boldsymbol{\omega} \right\rangle_{\boldsymbol{G},I} + \left\langle \boldsymbol{\omega} \cdot \frac{\partial}{\partial \boldsymbol{x}} \ln P(\boldsymbol{x}|I) \right\rangle_{\boldsymbol{G},I} = \sum_{j=1}^{m} \left[\frac{\partial}{\partial G_j} \left\langle \frac{\partial g_j}{\partial \boldsymbol{x}} \right\rangle_{\boldsymbol{G},I} + \left\langle \frac{\partial g_j}{\partial \boldsymbol{x}} \right\rangle_{\boldsymbol{G},I} \frac{\partial}{\partial G_j} \ln P(\boldsymbol{G}|I) \right] \tag{4}$$

that not only contains the CVT but also the fluctuation-dissipation theorem. In this work we show the derivation of this identity and several examples of its use, from constructing configurational temperature estimators under constraints [5], performing coordinate transformations, computing probability distributions of sums of variables and even the calculation of densities of states of model systems.

Acknowledgments

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II International Workshop on Statistical Physics
Antofagasta, Chile, December 01-03, 2021 Topic(s): Statistical Mechanics and Fundamentals

Curvature of Ruppeiner geometry in different parameterizations

Garcia-Ariza M. A.[1,*]

[1] *Arkansas State University campus Queretaro, Carretera Estatal 100 km 17.5, CP 76270, Colon, Qro., Mexico*
[*] mgarciaariza@astate.edu

Introduction

It is well known that thermodynamic fluctuations around a state of maximum entropy $\bar{x}$ can be described from a Riemannian-geometric standpoint, known as Ruppeiner geometry [1], whose metric tensor g is locally given by

$$g_{ij}(\bar{x}|\theta) = -\frac{\partial^2 S}{\partial x^i \partial x^j}(\bar{x}|\theta),$$

where the x^i are the so-called "natural variables" of the entropy S of the system under the constraints determined by the parameters θ. This approach seeks to lay out physical phenomena, especially those related to critical behavior, microscopic interactions, and phase transitions, in terms of the curvature of the manifold of equilibrium states. In particular, for a system with two fluctuating variables, the latter is given by $R = -\det(h)/(2\det(g)$, where

$$h = \begin{pmatrix} g_{11} & g_{12} & g_{22} \\ \partial_1 g_{11} & \partial_1 g_{12} & \partial_1 g_{22} \\ \partial_2 g_{11} & \partial_2 g_{12} & \partial_2 g_{22} \end{pmatrix}.$$

While this treatment of equilibrium thermodynamics has provided useful information in a variety of contexts [2-7], it is heavily reliant on coordinates, unlike the widespread geometric approaches to classical mechanics and general relativity. This brings about problems in the physical interpretation of curvature, as this work illustrates. Upon introducing a geometric notion of natural variables of entropy, I show that the curvatures of the submanifolds that are relevant in fluctuation theory depend on the manifold on which states fluctuate. This means that Ruppeiner geometry requires a preferred parameterization of the manifold of states to yield physically meaningful results. In order to reconcile Ruppeiner geometry with covariance, I hint at some possible amends that might yield a coordinate-independent fluctuation theory in a broader setting.

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Two-dimensional diffusion biased by a transverse gravitational force in an asymmetric channel

Ivan Pompa-García[1✉], Leonardo Dagdug[1,2§]

[1] *Universidad Autónoma Metropolitana - Iztapalapa. Ciudad de México, México*
[2] *National Institutes of Health, Shriver National Institute of Child Health and Human Development. MD, U.S.*
✉ivanjpg@xanum.uam.mx, §dll@xanum.uam.mx*

Introduction

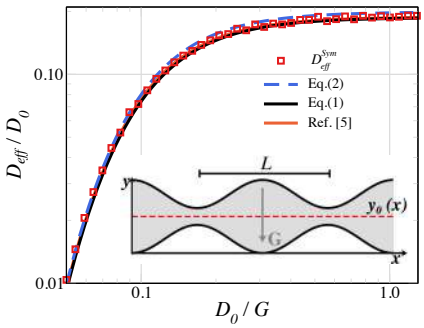
Using the projection method by Kalinay and Percus [3], we derive an effective diffusion coefficient for narrow channels that generalizes previously reported results. This is, a position-dependant diffusion coefficient for two-dimensional asymmetric channels under a transverse gravitational external field is obtained. The main result shown here as Eq. (1)

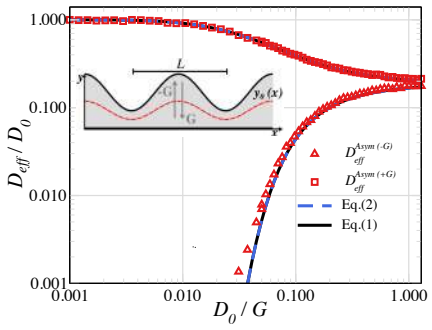
$$\begin{aligned}\frac{D(x)}{D_0} = 1 - \frac{w'^2(x)}{4\sinh^2\left[\frac{1}{2}gw(x)\right]} \times \left\{1+\cosh^2\left[\frac{1}{2}gw(x)\right] - gw(x)\coth\left[\frac{1}{2}gw(x)\right]\right\} \\ - y_0'(x)\left\{y_0'(x) - w'(x)\coth\left[\frac{1}{2}gw(x)\right] + \frac{1}{2}gw(x)w'(x)\,\mathrm{csch}^2\left[\frac{1}{2}gw(x)\right]\right\},\end{aligned} \tag{1}$$

contains the well-known previous results for symmetric channels with external gravitational force presented by Kalinay [4], as well as asymmetrical cases where the transverse field goes to zero. Also, found coefficient can be approximately written as an interpolation formula as proposed initially by Reguera and Rubi [2]

$$D_\eta(x) = \frac{D_0}{\left[1+\frac{1}{4}w'^2(x)\right]^\eta}, \tag{2}$$

where the constant transverse force, assymetry, and spatial confinement can be encoded in η. Eqn. (2) can be used to recover preceding results as well. Finally, the excellent agreement of our equations with Brownian dynamics simulations is shown for both cases. Symmetric channels that are compared, also with previously obtained results by another method [5]. And for asymmetric channel boundaries that are new promising results that could be used to control effective diffusion and particle separation. These simulations results are shown in the figures below and the channels used for each one are depicted in the inset images.





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*Corresponding Author

Complexity and disequilibrium in systems with long-range interactions

Boris Atenas†, Flavia Pennini, Sergio Curilef *
Universidad Católica del Norte, Antofagasta
†batenas@ucn.cl, fpennini@ucn.cl, *scurilef@ucn.cl

The systems can be classified according to their interactions in short and long-range. Free particles and systems with short-range interactions are simple; however, in systems with long-range interactions the emergence of chaos and others properties as the lack of additivity and extensivity, anomalous diffusion, etc., results in phenomena with more complexity [1,4]. Therefore, biological and atmospheric systems, neural networks, financial markets, etc. are examples of complex systems.
Disequilibrium (D) and complexity (C) are two statistical quantities introduced by [4] Lopez-Ruiz-Mancini-Calbet (LMC) to measure the degree of complexity of a system being the limiting cases, the ideal gas and perfect crystal, both with zero complexity. The above quantities are defined as,

$$D = \int \rho^2(x)dx, \quad C = DS. \tag{1}$$

In the current abstract, we compute the complexity and the disequilibrium as a function of the temperature in the so-called d-HMF model, introduced recently [5], representing a typical system with long-range interactions. For high temperatures, we obtain the expected results for these quantities; however, the main limitation lies in the theoretical description for low temperature due to the classical nature of this Hamiltonian.

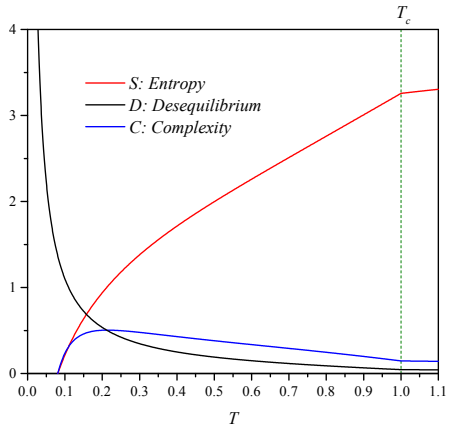

Figura 1: *Entropy, Disequilibrium and Complexity of the d-HMF model as a function of temperature T.*

Acknowledgment
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Information measures in bistable potentials

M. P. Escobar,* F. Pennini † and S. Curilef.§

Departamento de Física, Universidad Católica del Norte, Av. Angamos 0610, Antofagasta, Chile.
*mea005@alumnos.ucn.cl, †fpennini@ucn.cl, §scurilef@ucn.cl

Introduction

In this work, a study of the behavior of information measurements in two types of potential bistables is carried out, the Razavy bistable potential[1] and the square bistable potential, to contribute new knowledge to the investigation of statistical quantities in quantum systems. In the Razavy bistable potential study, we corroborate the invariance under replication of the probability density for the stationary states proposed by López-Ruiz and Sañudo[2] in the framework of the analysis of the infinite quantum square well. In addition, in the research of the square bistable potential, we ask the implications of the quantum tunneling in the description of the information, disequilibrium, and complexity measurements, which, as expected according on this particular phenomenon, shows us that the probability density is more concentrated in the wells than within the potential barrier.

Methodology and results

Sommerfeld's analytical method obtains the lowest energy levels, which allows us to compute the information, disequilibrium, and statistical complexity measures. Such a calculation depends on the probability density of the lowest eigen-energies of the Hamiltonian, including the corresponding bistable potential. In the square bistable potential study, we find analytical solutions for a one-dimensional symmetric potential, solving the Schrödinger equation for each of the regions observed in the system. We obtain the energy quantization condition for each of these solutions, which calculates the allowed values for the lowest energy levels. We determine the expression of the time-dependent wavefunction for the system ground state and, with it, the time-dependent probability density. We compute the time evolution of the information measurements during a period of oscillation for different parameter values of the system.

We numerically check the invariance under replication, which shows us that the information measures remain constant for the lowest energy levels. In the study of a particle confined in the square bistable potential, we note that the temporal evolution of the information measures behaves as a periodic function that takes extreme values when the particle is in one of the wells and when it crosses the potential barrier.

Acknowledgments

We would like to acknowledge FONDECYT-Chile, grant 1181558, for partial financial support.

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Many coupled springs as a discrete model for a classical string

Benny Nogales[1*] and Sergio Curilef[2†]

[1] *Universidad Mayor de San Simón, Av. Ballivián esquina Reza ♯591, Cochabamba, Bolivia*
[2] *Universidad Católica del Norte, Av. Angamos 0610, Antofagasta, Chile*

[*] bennynoga86@gmail.com, [†] scurilef@ucn.cl

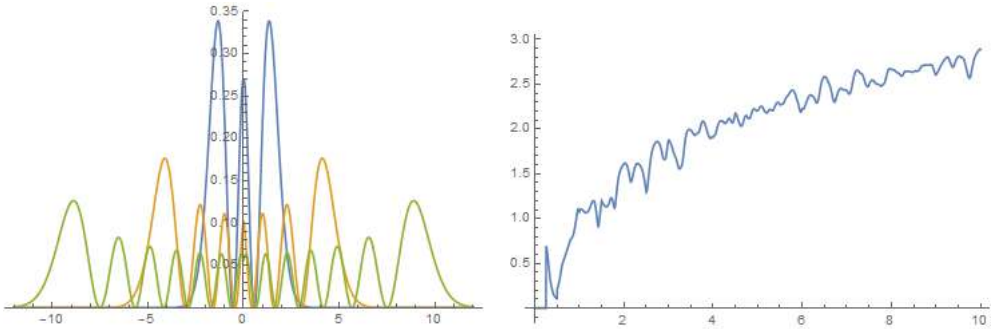
We present a study of longitudinal/transversal vibrations on a string using a discrete system of point masses equally separated as an approximation to model the string. This approximation considers that deviations from the equilibrium point are made by springs that get stretch and get compress[1]. Then, we write the equations of motion from a Lagrangian formulation and find the frequency dispersion ratio using plane wave solutions. Since the N equations of motion are coupled, we propose an expression for a wave packet by superposition. Finally, considering an impulse highly localized in the central point of the system as the initial condition of the problem, we describe how the wave packet expands in the string. If ϕ_l represents the l-th vibration, the Lagrangian formulation leads to the following expressions:

$$L = \frac{1}{2}\left(\sum_{l=-n}^{n} m\dot{\phi}_l^2 - \omega_0 \sum_{l=-n}^{n-1} (\phi_{l+1} - \phi_l)^2\right) \implies \ddot{\phi}_l - \omega_0^2((\phi_{l+1} - \phi_l) - (\phi_l - \phi_{l-1})) = 0.$$

Considering a plane-wave solution type, we obtain the frequency dispersion ratio given by $\omega(k) = 2\omega_0 \sin\left(\frac{ka}{2}\right)$. Thus, using a wave packet highly localized to the center of the string, we finally obtain

$$\theta_l(t) = \frac{1}{2\pi}\int_{-\pi}^{\pi} A_l(k) e^{i\omega t} d\left(\frac{ka}{2}\right) = J_{2l}(2\omega_0 t),$$

which corresponds to the amplitude time evolution of vibrations. From this expression, we can build the classical probability, which can be used for several applications in statistical physics and complexity.



(a) (b)

Figure 1: *(a) Classical probability* $\mathcal{P}_l(t) = J_{2l}(2\omega_0 t)^2 / \sum_l |J_{2l}(2\omega_0 t)|^2$ *as a function of the location* l*, for fixed times. (b) The evolution of the Information (Shannon Entropy)* $\mathcal{I}_S(t) = -\sum_l \mathcal{P}_l(t) \log \mathcal{P}_l(t)$ *of the system.*

Acknowledgment

We would like to thank the "Núcleo de Investigación No 2-Sistemas Complejos en Ciencia e Ingeniería-UCN-VRIDT 042/2020" for the scientific support.

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A Dissipative Particle Dynamics study of phase separating fluid mixtures with polymeric component on its interface

Dorothy Gogoi[1*], Awaneesh Singh[2,†], Sanjay Puri[1]

[1] *School of Physical Sciences, Jawaharlal Nehru University, New Delhi-110067, India.*
[2] *Department of Physics, IIT (BHU), Varanasi, Uttar Pradesh-221005, India.*

[*] dorothyjnu@gmail.com, [†] awaneesh.phy@itbhu.ac.in

Introduction

The process of phase separating multicomponent mixtures is of great interest to experimentalists and theorists alike due to its huge scientific and technological importance. At a high enough temperature ($T \geq T_c$), the mixture prefers to be in a homogenous or disordered state due to large contribution from entropy. When rapidly quenched below the critical temperature (T_c), the system becomes thermodynamically unstable and evolves towards the new equilibrium state by the formation and growth of domains of like particles [1,2]. It is now well-established that these coarsening dynamics is a scaling phenomenon, such that the quantities that characterize the dynamics, e.g., the two-point equal-time *correlation function* $C(r,t)$ and its Fourier transform, the *structure factor* $S(k,t)$ exhibits dynamical scaling form:

$$C(r,t) = g[r/l(t)], \tag{1}$$

$$S(k,t) = L(t)^d f[kl(t)]. \tag{2}$$

Here, $g(x)$ and $f(p)$ are the scaling functions independent of time, r is the separation between the spatial points, k is the magnitude of the wave vector and d is the system dimensionality. The average domain-size follows a unique and simple power-law dependence on time: $l(t) \sim t^{\phi}$ where ϕ is referred to as the growth exponent [3].

Model

We consider a ternary fluid mixture (ABC), where A and B are simple fluids and C is the polymeric component trying to reside on the interface of the fluid mixtures. The modeling of the mixture is done through Dissipative Particle Dynamics (DPD) [4,5] which naturally incorporates the hydrodynamics effect. The interacting particles in the system evolve in time by the integration of Newton's equation of motion, $m_i \frac{d\mathbf{v}_i}{dt} = \mathbf{f}_i$ where the effective force $\mathbf{f}_i(t)$ acting on each bead i consists of three pairwise additive forces: the conservative, dissipative and random forces. We do a comprehensive study of the system by varying the composition and various parameters of the polymeric component and try to find out what effect a polymeric component (C) on the interface of simple binary (AB) fluid may have on the phase separation kinetics.

Acknowledgement

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II International Workshop on Statistical Physics
Antofagasta, Chile, December 1-3, 2021 Topic(s): **Statistical mechanics and Fundamentals**

Bose-Einstein statistics for a finite number of particles

Pedro Pessoa[1*]
[1] *Physics Department, University at Albany (State University of New York)*
[*] ppessoa@albany.edu

Introduction

This talk will present the results of my recently published paper [1].

Bose-Einstein (BE) condensation is one of the most important examples of phase transitions (and the first one to be taught in most textbooks). This phase transition is identified by the fact that there is a low value of temperature, namely the critical temperature, for which (in the thermodynamic limit) the fraction of particles in the lowest energy state (ground state) and the specific heat are nonanalytical.

In this talk I will present a study of the grand canonical Bose-Einstein (BE) statistics for a finite number of particles in a general dispersion relation. In a finite number of particles not only the nonanalytical behaviour disappears but important qualitative diferences can be seen for a number of particles of the order of the ones observed experimentally in Refs.[2].

This is an important result for understanding the role of the thermodynamic limit in phase transitions and makes possible to further study BE statistics without relying neither on the thermodynamic limit nor on approximations near critical temperature.

The problem and results

The BE critical temperature is defined in terms of the expected number of particles N as

$$\beta_c = \left[\kappa \frac{\Gamma(\eta+1)}{N}\zeta(\eta+1)\right]^{\frac{1}{\eta+1}} ,$$

where ζ refers to the Riemann's zeta function, $\kappa^{\frac{1}{(\eta+1)}}$ is an unit of energy given by the system, and η is the density of states exponent given by the quantum system – $\eta = \frac{1}{2}$ for a gas trapped in a box and $\eta = 2$ for a harmonically trapped gas, calculation of η for others potentials can be found in [1].

The number of particles N and the internal energy U of a Bose gas are given, respectively by

$$N = \kappa \frac{\Gamma(\eta+1)}{\beta^{\eta+1}}\mathrm{Li}_{\eta+1}(\xi) + n_0 , \quad \text{where} \quad n_0 = \frac{\xi}{1-\xi}$$

$$U = \kappa \frac{\Gamma(\eta+2)}{\beta^{\eta+2}}\mathrm{Li}_{\eta+2}(\xi) .$$

Above β refers to the unit corrected inverse temperature, $\beta = \frac{1}{k_B T}$; ξ to the fugacity; ; and Li refers to the polylogarithm family of functions, $\mathrm{Li}_\varphi(y) = \sum_{k=1}^{\infty} \frac{y^k}{k^\varphi}$.

With the equations for N and U above, all thermodynamic quantities can be calculated exactly in terms of β and ξ, however for a adequate comparison with the critical temperature, it is necessary to obtain the thermodynamical quantities in terms of β and N. This is possible by defining $\xi(N,\beta)$ as the inverse of the expression for N above, Ref. [3] presents a method to calculate $\xi(N,\beta)$ numerically.

We will observe that this leads to several qualitative differences in the behaviour of the thermodynamic quantities that define phase transition for a number of particles ranging from $N = 10^2$ to $N = 10^7$ – which is the order of N in which the experiments reporting BE condensation were performed [2] – when compared to the thermodynamic limit.

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Density of states of a Three-state-Potts Coulomb Lattice Gas

Felipe Moreno[1*], Joaquín Peralta[1,†], Sergio Davis[2,1]

[1] *Departamento de Física, Facultad de Ciencias Exactas, Universidad Andres Bello, Santiago, Chile.*
[2] *Research Center on the Intersection in Plasma Physics, Matter and Complexity, P^2MC, Comisión Chilena de Energía Nuclear, Santiago, Chile.*

*f.moreno.munoz@gmail.com, †joaquin.peralta@unab.cl

Introduction

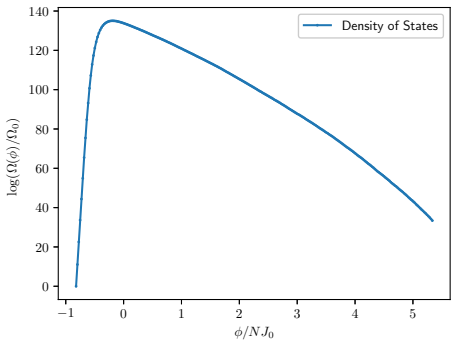
We present the computation of the configurational density of states (CDOS) of a three-state Potts model arising from the Coulomb lattice gas approximation. The Potts spin states $\sigma_i \in \{-1, 0, 1\}$ represent negative charge, absence of charge and positive charge, respectively, and in this way, the model captures some features of charged particles confined in a given geometry but introducing an underlying discrete lattice that makes it possible to optimize the computational cost of the simulation. We make use of the Wang-Landau algorithm [1] to compute the CDOS, and from it we compute the microcanonical and canonical caloric curves for the system. We start from the Coulomb interaction in a system composed of n particles

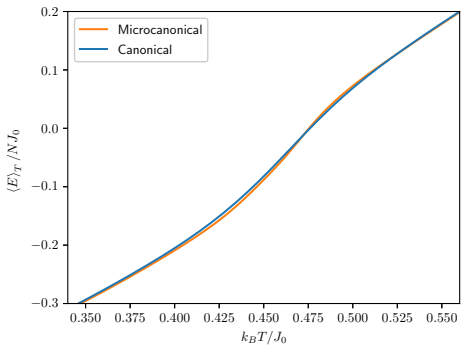
$$\Phi(\boldsymbol{r}_1, \ldots, \boldsymbol{r}_n) = \frac{1}{2} \sum_{i,j \neq i} \frac{q_i q_j}{4\pi\epsilon_0 |\boldsymbol{r}_i - \boldsymbol{r}_j|} \tag{1}$$

confined inside a sphere, that is, with $|\boldsymbol{r}_i| < R$, which can be transformed into a lattice-gas model by fixing N possible positions of the particles in a grid and only recording the charge $\sigma_i \in \{-1, 0, 1\}$ in a given site, for $i = 1, \ldots, N$. In this way we can write [2]

$$\Phi(\sigma_i, \ldots, \sigma_N) = \frac{J_0}{2} \sum_{i,j \neq i} \frac{\sigma_i \sigma_j}{r_{ij}} = \frac{1}{2} \sum_{i,j \neq i} J_{ij} \sigma_i \sigma_j \tag{2}$$

where $J_{ij} = J_0 / r_{ij}$ and $J_0 = e^2/(4\pi\epsilon_0 r_0)$ with r_0 the minimum distance between neighbors in the grid, and where we impose the conservation of charge by fixing $N = \sum_{i=1}^{n} |\sigma_i|$ and $\sum_{i=1}^{n} \sigma_i = 0$.





Acknowledgements

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II International Workshop on Statistical Physics
Antofagsta, Chile, 01-03 de diciembre de 2021 Topic(s): Statistical Mechanics and Fundamentals

A Lorentz invariant distribution for a relativistic gas

Evaldo M. F. Curado[1*,2], Carlos E. Cedeño [1], Ivano Damião Soares[1], Constantino Tsallis[1,2]
[1] *Afiliación A, Centro Brasileiro de Pesquisas Físicas, Rio de Janeiro, Brazil*
[2] *Afiliación B, National Institute for Science and Technology for Complex Systems, Rio de Janeiro, Brazil*
[*] evaldo@cbpf.br,

Abstract
In 1911, Jüttner proposed a generalization of the Maxwell-Boltzmann velocity distribution for a relativistic gas. Here, we want to discuss the Jüttner's probability density function (PDF). The volume in velocity space and consequently in momentum space is not flat in special relativity. The velocity space is curved and corresponds to the Lobachevsky space. As a result, this induces a different power for the Lorentz factor in the PDF, affecting Jüttner's normalization constant in one, two and three dimensions. Furthermore, Jüttner's distribution written in a more appropriate variable, the rapidity, presents a curvature change at the origin at a sufficiently high energy, which does not agree with the computational dynamics simulation of a relativistic gas. However, in at least in one dimension, the rapidity satisfies a simple additivity law, allowing us to obtain a new PDF whose curvature at the origin does not change for any energy value and which agrees with the computational dynamics simulation data.

Spin-Statistics Connection in the context of Very Special Relativity

Alessandro Santoni [1,†], Enrique Muñoz[1*], Benjamin Koch [1,2,○]

[1] *Pontificia Universidad Católica de Chile, Instituto de Física, Pontificia Universidad Católica de Chile*
[2] *Institute for Theoretical Physics, TU Wien, Wiedner Hauptstr. 8, A-1040 Vienna, Austria*

[†] asantoni@uc.cl [*] munozt@fis.puc.cl, [○] bkoch@puc.uc.cl

Introduction

In 2006, A. Cohen and L. Glashow presented for the first time the idea of *"Very Special Relativity"* (VSR), a theory supposing as spacetime simmetry no longer the full Lorentz group but a particular class of Lorentz's subgroups [1]. These subgroups have the special property of being enlarged to the full Lorentz group when adding some discrete simmetry, like CP invariance. Due to the already known small CP breaking effects in the Standard Model, VSR opened new possibilities for particle physics, in particular concerning different mechanisms for neutrino masses [2].
The aim of our work is to study the Spin-Statistics connection with a focus in the framework of VSR: as we know, in fact, the different proofs of the Spin-Statistics Theorem, starting from the historic one by Pauli [3], often relies in some way on Lorentz invariance and on locality of the Lagrangian terms, while both of these assumptions may be violated in VSR.
In the following, we will consider as VSR group the so-called $SIM(2)$, generated by the transformations $T_1 = K_X + J_Y$, $T_2 = K_Y - J_X$, K_Z and J_Z.

VSR Field Theories, CPT Invariance and Spin Statistics Connection

For an explicative purpose, we will present here a brief and pragmatic summary of the analysis in the case of the VSR fermion field theory

$$\mathcal{L}_f = \bar{\psi}(i\gamma^\mu\partial_\mu - M + i\frac{m^2}{2}\frac{n^\mu\gamma_\mu}{n\cdot\partial})\psi,$$

where n is the preferred VSR null vector breaking full Lorentz simmetry to $SIM(2)$ invariance. We will exploit the CPT simmetry of nature, which it's not broken by reducing to $SIM(2)$ invariance. Let's consider the notation [4] for the C, P and T transformations

$$P\psi(t,\vec{x})P = \eta_P\gamma^0\psi(t,-\vec{x})\ , \quad T\psi(t,\vec{x})T = -\eta_T\gamma^1\gamma^3\psi(-t,\vec{x})\ , \quad C\psi(t,\vec{x})C = -i(\bar{\psi}(t,\vec{x})\gamma^0\gamma^2)^T,$$

where the three η are arbitrary phases. Then, we can calculate the CPT transform of each term $\mathcal{L}_i(x)$ of the Lagrangian, verifying under which commutation fermion conditions we get $\mathcal{L}_i(-x)$ and therefore CPT invariance of the field action.
In particular, since under CPT $n_\mu \to -n_\mu$, $\partial_\mu \to -\partial_\mu$, having in the VSR term a fraction with n^μ in both numerator and denominator, its behaviour under CPT is effectively the same of the kinetic term, whose transformation is

$$\bar{\psi}(t,\vec{x})\gamma^\mu\partial_\mu\psi(t,\vec{x}) \to -(\gamma^0\gamma^5\psi(-t,-\vec{x}))^T\gamma^\mu\partial_\mu(\bar{\psi}(-t,-\vec{x})\gamma^0\gamma^5)^T = -(\bar{\psi}(-t,-\vec{x})\partial_\mu(\gamma^\mu)^T\psi(-t,-\vec{x}))^T,$$

which, as expected, implies rules of anticommutation for $\bar{\psi}$, ψ to get CPT invariance of the action. That implies the real possibility of writing, under the right assumptions, VSR field theories that respect, as in the above example, the Spin-Stastics Connection.

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On a general class of statistical complexity measures

Velimir M. Ilić[1*]
[1] *Mathematical Institute of the Serbian Academy of Sciences and Arts, Beograd, Serbia*

Introduction

In the past decades there was a plausible interest in the definition and characterization of the measures of certainty and information associated with probability distribution, as well as in the measures which characterize the distance between two distributions. In this line of research, axiomatic approaches are particularly powerful tools and could serve as an invaluable way to analyze the intriguing relationships between information and complexity. The most prominent one is based on generalized Shannon-Khinchin axioms [1], from which the class of strongly pseudo-additive entropies can be derived [1], which includes Sharma-Mittal, Rényi, Havrda-Charvát-Tsallis and Gaussian entropies, as well as Massi and Jensen-Pazuki-Pruessner-Tempesta entropies, while the InforCer measures and corresponding axiomatic system introduced in [2] define an even more general family of measures for the quantification of information, certainty and inaccuracy. This leads to a variety of generalized divergences measures between two distributions (see [3] and references therein) and has strong implications for the characterization of the statistical complexity measures, which represent an interplay between order and disorder levels of complex system, where generalized entropies are usually used for the disorder measures, while the order is described using generalized divergence measures. In the past, different definitions of statistical complexity based on different choices of the entropy and divergence measures were introduced, but there is no a general agreement which one is preferable [4-7], due to the fact that it has still not been accepted what is a set of basic properties which complexity measure should satisfy.

The problem and results

In this work we provide a general treatment of the statistical complexity measures and a detailed analysis of existing measures, starting from a set of ineluctable properties, which are stated as axioms, by which the statistical complexity measure has to be non-negative, decomposable, equal to zero, in the case of a simple system, invariant under rescaling and replication and to preserve consistency of discrete and continuous cases. We propose a general class of statistical complexity measures, which depends on a parameter q and satisfy all of the properties. Unlike the previous approaches, both order and disorder parts are defined using generalized divergence measures. This approach ensures that proposed measures have well defined limit when a discrete system tends to a continuous, so that the definitions of the complexity in these two cases are consistent. In addition, unlike the previously considered measures, the proposed complexity measure can be extensive or non-extensive by changing the value of the parameter q.

Acknowledgment

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Physics of Evolution and Structure of Matter

V.M. Somsikov
Al-Farabi Kazakh National University, Almaty, 050040, Kazakhstan
vmsoms@rambler.ru

Introduction

The work is devoted to the development of a method for describing the dynamics of structured bodies (SB). The main advantage of the method is that it opens up the possibility of a deterministic description of the processes of evolution of bodies within the framework of the fundamental laws of physics. This possibility is due to the use in it of the equation of motion of the SB, obtained taking into account the relationship of its dynamics, as a whole, with the internal dynamics of the elements of the SB. The derivation of the SB equation of motion is based on the principle of symmetry dualism. According to this principle, the dynamics of the SB is determined by both the internal symmetries of the SB and the symmetry of the space. In accordance with the principle of symmetry dualism, the derivation of the SB of motion equation was carried out directly from the condition of invariance of the total energy of the SB. Moreover, the total energy is the sum of the energy of motion of the SB and its internal energy. The representation of the total energy of the SB is realized in the space of independent micro- and macro-variables, which determine the energy in the form of the sum of its internal energy and the energy of motion, respectively. The derivation of the equation of motion of the SB from energy made it possible to avoid the limitations that exclude the possibility of describing dissipative processes. These restrictions arise in the Lagrange and Hamilton equations due to the use of the conditions of holonomic constraints and the potentiality of collective forces in their derivations. These conditions lead to the reversibility of the equations of motion of the systems. The SB motion equation takes into account the work of dissipative friction forces that determine the transformation of motion energy into internal energy. This made it possible to introduce in the framework of classical mechanics the concept of D-entropy, defined as the relative increase in the internal energy of the SB due to its energy of motion. D-entropy is convenient for analyzing evolutionary processes in moving interacting bodies, for example, for objects in the Universe. In general, the equation of motion of the SB made it possible to reconcile the reversibility of the motion of the MP, on the one hand, and the irreversibility of thermodynamic processes, on the other hand.

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II International Workshop on Statistical Physics
Antofagasta, Chile, December 01-03, 2021 Topic(s): **Systems out of equilibrium**

A spatially extended mean field approach for flocking phenomena

Daniel Escaff*

Universidad De Los Andes, Chile.
*escaffnetmail@yahoo.com

A Kuramoto-type approach to address flocking phenomena is presented. First, we analyze a simple generalization of the Kuramoto model for interacting active particles, which is able to show the flocking transition (the emergence of coordinated movements in a group of interacting self-propelled agents). In the case of all-to-all interaction, the proposed model reduces to the Kuramoto model for phase synchronization of identical motionless noisy oscillators. In general, the nature of this non-equilibrium phase transition depends of the range of interaction between the particles. Namely, for small range of interaction the transition is first order, while for a larger range of interaction it is a second order transition. Moreover, for larger interaction ranges, the system exhibits the same features that in the case of all-to-all interaction, showing a spatially homogeneous flux when flocking phenomenon has emerged. While, for lower interaction ranges, the flocking transition is characterized by cluster formation. We compute the phase diagram of the model, where we distinguish three phases as a function of the range of interaction and the effective coupling strength: a disordered phase; a spatially homogeneous flocking phase; and cluster-flocking phase. Then, we present a general discussion about the applicability of this way of modeling to more realistic and general situations, ending with a brief presentation of a second example (a second model with conservative interaction) where the flocking transition may be studied within the framework that we are proposing.

Non-equilibrium statistical mechanics tool for the study of space plasmas; The Ehrenfest procedure in Earth's radiation belts and Superstatistics in magnetized plasma.

Abiam Tamburrini[1,*], Sergio Davis[2,3,†], Pablo S. Moya[1,‡]

[1] *Departamento de Física, Facultad de Ciencias, Universidad de Chile*
[2] *Comisión Chilena de Energía Nuclear, Casilla 188-D,Santiago*
[3] *Departamento de Física, Facultad de Ciencias Exactas, Universidad Andrés Bello*

[*] abiam.tamburrini@ug.uchile.cl, [†] sdavis@cchen.cl, [‡] pablo.moya@uchile.cl

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An interesting problem in plasma physics, when approached from the point of view of non-equilibrium Statistical Mechanics, is to obtain properties of collisionless plasmas, through the Vlasov Equation [1]. From Classical and Statistical Mechanics, this equation corresponds to a fluid version of the Liouville theorem for the phase space density, when the Hamiltonian describes electromagnetic interactions and the effect of Coulomb collisions can be neglected. Even though the Vlasov Equation is a well-known theoretical approach in plasma physics, the obtention of solutions is a very difficult task, usually addressed using perturbative methods. In this work we present two ways of approaching the problem. First, from the point of view of obtaining macroscopic properties of the system. Through theoretical analysis and numerical calculations we show that starting from the Vlasov equation, and using a classical analog of Ehrenfest theorem [2,3], it is possible to derive relations for the expectation values of time-dependent observables [4]. For this case, considering charged particles trapped in a magnetic field dipole, the three adiabatic invariants are studied indirectly. Considering expression for pitch angle, average radius and magnetic moment we found dynamical equations that we contrast using test-particle simulations. Second, considering the origin of empirical distributions in modeling space plasma phenomena is not a settled issue, we also approach the problem from a microscopic point of view. We start from a linear approximation of the Vlasov Equation, and apply Superstatistics considerations to explore its scope and possible interpretation on dispersion relations for a magnetized plasma, extending Ourabah's previous analysis on electrostatic plasma waves [5].

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Many-Body Master Equation for Interacting Brownian Particles

A. G. Deaker[1,*], M. W. Jack[1,†], N. J. López-Alamilla[1], K. J. Challis[2]

[1] *Department of Physics, University of Otago, P. O. Box 56, Dunedin 9054, New Zealand*
[2] *Scion, 49 Sala Street, Rotorua 3046, New Zealand*

[*] deaad494@student.otago.ac.nz, [†] michael.jack@otago.ac.nz

The Asymmetric Simple Exclusion Process (ASEP) is a paradigmatic model of an interacting non-equilibrium system [1]. It describes Brownian particles occupying discrete lattice sites with strong inter-particle interactions that limits site occupation to one or zero particles. The ASEP model has been successfully used to describe a wide variety of non-equilibrium phenomena including stepping molecular motors [1]. Despite its utility, it is not clear how to generalise this model to the cases of multiple site occupation or weak interactions.

In this talk we present a many-body master equation that generalises the ASEP model to the case of multiple site occupation. We formulate this master equation by exploiting boson creation and annihilation operator methods from many-body quantum mechanics. These techniques have been shown to also apply to indistinguishable classical particles [2]. This master equation describes discrete hopping between lattice sites and repulsive interactions between particles. Due to the similarity to quantum systems, we can apply well-known numerical and analytical techniques to solve for the non-equilibrium steady state. We present results for the steady-state drift and number fluctuations of which have been calculated using the Gutzwiller approximation [3] and operator identities. For example, the steady-state drift for a range of interaction strengths from non-interacting to strongly interacting is shown in Figure 1. In the limits of strong repulsive interactions and less than one particle per site, this master equation reduces to the ASEP model [4].

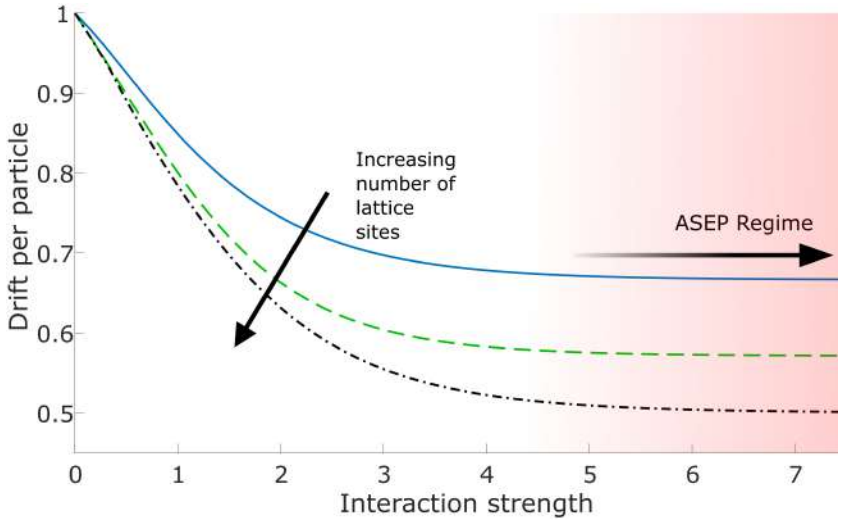

Figure 1: *Steady-state drift per particle for a density of 0.5 and (solid) a lattice with 4 sites, (dashed) a lattice with 8 sites and (dash-dotted) Gutzwiller approximation.*

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A self-organized critical system under the influence of turbulent motion of the environment

Aleksandr Yu. Luchin[1,*], Polina I. Kakin[1,‡], Nikolay V. Antonov[1,†],

[1] *Department of High Energy and Elementary Particles Physics, Saint Petersburg State University, Universitetskaya Emb. 7/9, St. Petersburg, 199034, Russia*

[*] luhsah@mail.ru, [‡] p.kakin@spbu.ru, [†] n.antonov@spbu.ru

Introduction

The phenomenon of self-organized criticality (SOC) consists in the emergence of scaling in open non-equilibrium systems with dissipative transport [1,2]. Unlike equilibrium systems that arrive at critical states when control parameters approach their critical values [3], systems with SOC evolve to critical states due to their intrinsic dynamics. Such systems are widespread in nature with SOC being observed in physical, biological, economic and social systems [2].

Critical behavior of stochastic system can be drastically affected by turbulent motion of the environment (see, e.g., [4]), thus, it is important to study the motion influence on systems with SOC.

In this report, we present a field theoretic renormalization group analysis of the continuous anisotropic model of SOC introduced in [5] (a "running sandpile") coupled to the stochastic Navier–Stokes equation [6,7]. The latter describes turbulent motion of the environment; the random force correlator for the equation is chosen in the form that includes two terms. The first one is local and corresponds to shaking the fluid as a whole while the second one models "pure" turbulence.

Methodology

We constructed a field theory equivalent to the stochastic problem and performed analysis of ultraviolet divergences that shown that the theory is renormalizable. In order to find renormalization constants, Feynman diagrams related to diverging Green's functions were calculated to the first order of double expansion in two small parameters (one-loop approximation). Then the special case where SOC is excluded was considered. For this case, three regimes of critical behavior were found; corresponding critical exponents were calculated. Next step of the research is finding fixed points of the renormalization group equation for the problem where both turbulence and SOC are included equally because infrared attractive fixed points correspond to regimes of critical behavior and determine related critical exponents. This work is in progress.

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Run-and-tumble bacterial under chemical gradients

Andrea Villa-Torrealba[1,*], Rodrigo Soto[1,†]

[1] *Departamento de Física, Facultad de Ciencias Físicas y Matemáticas, Universidad de Chile, Santiago, Chile*
[*] aavillat@gmail.com, [†] rsoto@dfi.uchile.cl

Introduction

Chemotaxis is the ability of motile microorganisms to migrate along a chemical gradient. For *Escherichia coli*, this mechanism is a topic to study since not only expresses metabolic preferences, but also reveals bias in its motility due to the environment. Their motion is modeled as a run-and-tumble dynamics. In fact, their flagella can rotate and propel the cell body in a "run" mode which can suddenly terminate whenever some of them reverse direction called "tumble". The bacteria chemotaxis is achieved by modulating the tumble rate with changes in the concentration of the ligand. If the concentration increases over time the bacteria tend to keep moving in a straight line, tumbling less frequently. If this remains constant the bacteria return at the same tumbling rate.

The chemotaxis network of *E. coli*, has rationalized into a mathematical model where the tumbling rate depends on the concentration inside the bacterial body of the CheA protein, a, through the relationship $\nu(a) = \nu_0 e^{\chi a}$ [1]. Furthermore, the variable a evolves depending on the local value of the ligand concentration l and it is regulated by the methylation of CheB, m. In this sense, the protein concentrations are described by the Langevin equations [2]

$$\dot{a} = -\frac{1}{\tau_a}(a - \alpha m + \beta l) + \zeta_a, \tag{1}$$

$$\dot{m} = -\frac{1}{\tau_m} a + \zeta_m, \tag{2}$$

where ζ_a and ζ_m are white noise terms. $\tau_a \ll \tau_m$ which indicates that the relaxation of a to its average value is much faster than the m. α and β are the coupling constants of the m and l respectively.

In this work we use the tools of kinetic theory to make a quantitative analysis of the presented chemotaxis model. We derive the average bacterial current under steady and unsteady regimes and analyze how the bacteria are spatially distributed. Let us first consider the simple case of a uniform concentration gradient and a first linear response regime was obtained, while for large gradients the current saturates to the maximum possible value. From the linear regime, a static motility was obtained, which we give explicitly in terms of the model parameters. In addition, it was possible to determine how long it takes for the system to reach steady state when, suddenly, a chemotactic signal is established. For chemical signals, with spatio-temporal variation, we study how the population responds by analyzing the patterns that appear in the bacterial density.

Acknowledgments

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Langevin equation for the collective degrees of freedom of a binary astrophysical system

Claudio Michea,* and L. Velazquez.§

Departamento de Física, Universidad Católica del Norte, Av. Angamos 0610, Antofagasta, Chile.
*claudio.ramirez@ce.ucn.cl, §lvelazquez@ucn.cl

Introduction

This proposal seeks to understand the thermodynamic equilibrium between two astrophysical systems that are gravitationally bounded as a binary system. According to Thirring's condition [1], a system with negative heat capacity $C_1 < 0$ can be in equilibrium with other system with heat capacity $C_2 > 0$, provided that $C_2 < |C_1|$. Since astrophysical systems commonly exhibit negative heat capacities [2], one can naively expect that thermodynamic equilibrium is not possible for a binary system. Nevertheless, a binary astrophysical system behaves as a three-coupled subsystems by decomposing its description in terms of internal (A,B) and collective (C) degrees of freedom (see Figure 1).

$$H = H_A + H_B + H_C + W_{AB}. \tag{1}$$

In the framework of the quadrupole approximation:

$$H_C + W_{AB} = \frac{1}{2\mu}\mathbf{P}^2 - \frac{\alpha}{|\mathbf{R}|} - \frac{G}{|\mathbf{R}|^3}\left[M_A Q_B + M_B Q_A\right], \tag{2}$$

equilibrium between the subsystems (A,B) may be possible when the collective degrees of freedom behaves as a low dimensional system with positive heat capacity. Here, $\mu = M_A M_B/(M_A + M_B)$ is the reduced mass and $\alpha = GM_AM_B$ the gravitational coupling. Our interest is focused on a heuristic derivation of some sort of a Langevin equation that captures the dynamics of the collective degrees of freedom in *astrophysical thermal contact* with the internal degrees of freedom of each astrophysical system within the quadrupole approximation. Apparently, this model appears as an astrophysical analogue of a brownian particle coupled to two heat baths such as [3].

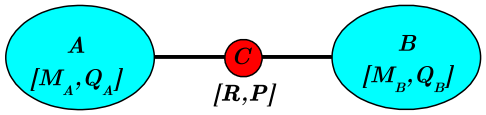

Figure 1: *Effective decomposition of a binary system into three-coupled subsystems in terms of the internal and collective degrees of freedom in the framework of quadrupolar approximation.*

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Multiple metastable states in an off-lattice Potts model

Constanza Farías[1*], Sergio Davis [2,†]

[1] *Departamento de Física, Facultad de Ciencias Exactas, Universidad Andres Bello. Sazié 2212, Santiago, Chile.*
[2] *Comisión Chilena de Energía Nuclear, Casilla 188-D, Santiago, Chile.*

[*] m.fariasparra@uandresbello.edu

Introduction

The interactions between a group of components are commonly studied in several fields of science including condensed matter physics[1], neuroscience[2], social science[3], percolation theory[4], among many others, using the methods of thermodynamics and statistical mechanics. In these types of systems we usually work with spin models, such as the Ising model and its generalization, the Potts model [5], have been proposed to describe correlation between individual sites. Furthermore, it is well known that in spin systems the phenomenon of metastability (a quasi-stable state of a dynamical system) can be observed [6, 7] and is common the use of the microcanonical ensemble. Metastable states are usually observed in thermodynamics states like superheated solids and supercooled liquids as also outside these kind of states, that is the case of complex systems where they are extremely sensitive to noise or perturbations. However, the phenomenon of metastability plays an important role in some basic processes of life, like the study of the enviroment as well as phenotypic difference in genetics [8].

In this work we study the properties of the recently proposed off-lattice, two-dimensional Potts model [Eur. Phys. J. B **87**, 78 (2014)] , a opinion model in social groups that takes into account how internal and external opinions may agree with each other. This model is described by a Hamiltonian obtained by a maximum entropy inference procedure. We performed microcanonical and canonical Monte Carlo simulations of the first-order phase transition in the model, revealing a caloric curve with metastable regions.

Furthermore, we report a "switching" behavior between multiple metastable states. We also note that the thermodynamics of the model has striking similarities with systems having long-range interactions, even though the interactions are short-ranged.

Acknowledgments

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Quantum transport across a slab of a type I Weyl semimetal with a uniform concentration of torsion dislocation defects

Daniel Bonilla[1*] and Enrique Muñoz[1,2,†]

[1] *Physics Institute, Pontificia Universidad Católica de Chile.*

[2] *Research Center for Nanotechnology and Advanced Materials, CIEN-UC, Pontificia Universidad Católica de Chile.*

[*] dabonilla@uc.cl, [†] munozt@fis.puc.cl

Introduction

Weyl semimetals (WSMs) are examples of three-dimensional gapless topological materials, in which the conduction and valence bands touch each other in an even number of points with linear dispersion, referred to as Weyl nodes. These nodes are protected from being gapped because they are monopolar sources of Berry curvature, and hence their charge (chirality) is a topological invariant [1]. In the vicinity of these nodes, low-energy conducting states can be described as Weyl fermions. While Type I WSMs fully respect Lorentz covariance, such condition is not satisfied in Type II WSMs. From the theoretical perspective, it has been proposed that different sorts of elastic strains can be modeled as gauge fields in WSMs [2]. It has been studied the effects of strain and magnetic field on the electronic and the thermoelectric transport properties of WSMs using the Landauer ballistic formalism in combination with the quantum mechanical scattering cross-sections[3].

The problem and results

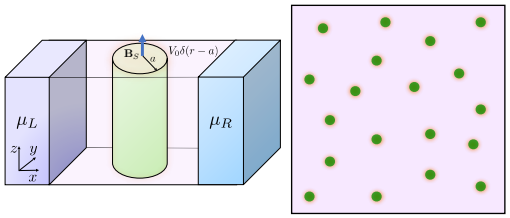

Figure 1: *Pictorial description of the system: (a) a single cylindrical defect and (b) a top view of the WSM slab with the uniform and diluted concentration of cylindrical defects.*

This work focuses on the transport properties of a WSM slab of volume L^3 with a uniform and diluted concentration of impurities created by applying mechanical torsion. Every single impurity consists of a cylindrical strip of WSM of radius $a \ll L$ under mechanical torsion represented by a gauge field, with a Repulsive Delta-Shell potential (RDSP) on its surface to represent the lattice mismatch due to strain. See Figure 1 for a pictorial representation. The single defect's scattering problem is known, and we can express the cross-section in terms of the phase shifts $\delta_m(k)$. In this work, we employ the Kubo formula for conductivity. We derive an expression for the conductivity across a cubic slab of WSM (with a concentration n_i of dislocation impurities per unit area) calculating the relaxation time as a function of the scattering phase shifts. The result is an isotropic conductivity tensor whose magnitude at finite temperature is

$$\sigma(T) = -\frac{2}{3\pi^2 n_i}\left(\frac{k_B T}{\hbar v_F}\right)^2\left(\frac{e^2 k_F}{\hbar}\right)\frac{\mathrm{Li}_2\left(-e^{\frac{\hbar v_F k_F}{k_B T}}\right)}{\sum_m \sin^2\left[\delta_m(k_F)-\delta_{m-1}(k_F)\right]}.$$

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II International Workshop on Statistical Physics
Antofagasta, Chile, December 01-03, 2021 Topic(s): **Systems out of equilibrium**

Mean-field interactions in evolutionary spatial games

Dmitriy Antonov[1], Evgeni Burovski[1,§] and Lev Shchur[1,2]
[1] *HSE University, 101000 Moscow, Russia.*
[2] *Landau Institute for Theoretical Physics, 142432 Chernogolovka, Russia.*
[§] evgeny.burovskiy@gmail.com,

Abstract

We introduce a mean-field term to an evolutionary spatial game model. Namely, we consider the game of Nowak and May, based on the Prisoner's dilemma, and augment the game rules by a self-consistent mean-field term. This way, an agent operates based on local information from its neighbors and non-local information via the mean-field coupling. We simulate the model and construct the steady-state phase diagram, which shows significant new features due to the mean-field term: while for the game of Nowak and May, steady states are characterized by a constant mean density of cooperators, the mean-field game contains steady states with a continuous dependence of the density on the payoff parameter. Moreover, the mean-field term changes the nature of transitions from discontinuous jumps in the steady-state density to jumps in the first derivative. The main effects are observed for stationary steady states, which are parametrically close to chaotic states: the mean-field coupling drives such stationary states into spatial chaos. Our approach can be readily generalized to a broad class of spatial evolutionary games with deterministic and stochastic decision rules [1].

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Quantum battery at the verge of a phase transition

Felipe Barra[1,*], Karen V. Hovhannisyan[2,3], Alberto Imparato[4]

[1] *Departamento de Física, Facultad de Ciencias Físicas y Matemáticas, Universidad de Chile, Santiago, Chile*
[2] *Institute of Physics and Astronomy, University of Potsdam, 14476 Potsdam, Germany*
[3] *The Abdus Salam International Centre for Theoretical Physics (ICTP), Strada Costiera 11, 34151 Trieste, Italy*
[4] *Department of Physics and Astronomy, Aarhus University, Ny Munkegade 120, 8000 Aarhus, Denmark*
[*] fbarra@dfi.uchile.cl

Introduction

The reduced state of a system strongly coupled to a bath is, in general, an athermal state. Starting from that observation, we introduce and study a battery–charger quantum device [1] storing energy in thermal equilibrium or a ground state [2]. The device operates in a cycle with four stages: the equilibrium storage stage is interrupted by disconnecting the battery from the charger, then work is extracted from the battery, and then the battery is reconnected with the charger; finally, the system is brought back to equilibrium. We study the case where the battery and charger together comprise a spin-1/2 Ising chain [3]. We show that the figures of merit—the extracted energy and the thermodynamic efficiency—can be enhanced by operating the cycle close to the quantum phase transition point. When the battery is just a single spin, we find that the output work and efficiency show a scaling behavior at criticality and derive the corresponding critical exponents. We found equivalent operations from the perspective of the battery, with different energetic costs for the cycle when the coupling term does not commute with the bare Hamiltonian, and we use this purely quantum leverage to optimize the device's performance.

Acknowledgments

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II International Workshop on Statistical Physics
Antofagasta, Chile, December 01-03, 2021 Topic(s): **Systems out of equilibrium**

On Turbulence and its Relation With Kappa Distributions: a Langevin Approach

Iván Gallo-Mendez,* and Pablo S. Moya.§
*§ *Departamento de Física, Facultad de Ciencias, Universidad de Chile*
* 1ggallo2009@gmail.com,

Abstract

Considering a coupled map lattice model [1] we analyze the relationship between turbulent cascades on the spatial scale and Kappa-like distributions representing the velocity probability distributions of eddies at different scales. We generate the steady-state velocity distribution of the fluid at each scale k and show that the generated distributions are well fitted by Kappa-like distributions. We observe a robust scaling relationship between the κ parameter, the scale, and the Reynolds number of the system, R_e. Our results show that there is a closed scaling relation between the level of turbulence and the κ parameter; namely $\kappa \sim R_e k^{5/3}$. Furthermore, we also consider skew velocity distributions that usually appear in turbulent systems driven by a chaotic forcing. We consider the Ulam map noise in the lattice and fit them with $\kappa\delta$-distributions (Beck's distribution) [2]. We characterize the relation between κ and δ parameters and focus on possible physical interpretation of skewness. We expect these results to be useful to characterize turbulence in different contexts, and our numerical predictions to be tested by observations and experimental setups [3,4].

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II International Workshop on Statistical Physics
Antofagasta, Chile, December 01-03, 2021 Topic(s): **Systems out of equilibrium**

Thermodynamics of structure-forming systems

Jan Korbel,* Simon David Lindner, Rudolf Hanel and Stefan Thurner.
Medical University of Vienna, Austria.
*jan.korbel@meduniwien.ac.at

Abstract

Structure-forming systems are ubiquitous in nature, ranging from atoms building molecules to self-assembly of colloidal amphibolic particles. The understanding of the underlying thermodynamics of such systems remains an important problem. Here, we derive the entropy for structure-forming systems that differs from Boltzmann-Gibbs entropy by a term that explicitly captures clustered states. For large systems and low concentrations the approach is equivalent to the grand-canonical ensemble; for small systems we find significant deviations. We derive the detailed fluctuation theorem and Crooks' work fluctuation theorem for structure-forming systems. The connection to the theory of particle self-assembly is discussed. We apply the results to several physical systems. We present the phase diagram for patchy particles described by the Kern-Frenkel potential. We show that the Curie-Weiss model with molecule structures exhibits a first-order phase transition.

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The surprising effectiveness of linear response theory for optimal protocols

Lucas Kamizaki[1*], Thalyta Martins[1,†], Sérgio R. Muniz[1]
[1] *Instituto de Física de São Carlos, Universidade de São Paulo (IFSC-USP)*
[*] lucas.kamizaki@usp.br, [†] thalyta@usp.br

Introduction

Sekimoto [1] showed that work and heat can be associated with individual trajectories of a Brownian particle. Thus, at the microscopic level, work is a stochastic quantity, and each trajectory leads to a different result when a control parameter changes from an initial to a final state. Therefore, after many realizations of an experiment, one gets a distribution of values of work. The average work is larger or equal to the free energy difference of the system, being equal only for infinitely long processes. In this context, an optimal protocol has the lowest average work for a finite time process. Here, we show that optimal protocols found using linear response theory (LRT) can perform well even outside its regime of validity.

The problem and results

There are a few analytic expressions of the optimal protocol for paradigmatic examples: time-dependent stiffness and time-dependent center position, in the overdamped limit, of a harmonic potential [2]. Nonetheless, optimal protocols are hard to find in general. A natural first approximation is to study weak processes, and the LRT method allows one to construct such protocols [3]. Its main advantage is that it is a phenomenological method, i.e., it does not require knowledge about the microscopic properties of the medium. Therefore, it is much simpler, but it is typically limited to a narrow range of values. Here, we present numerical studies of LRT applied to an optically trapped colloidal particle in dynamically controlled optical tweezers. We show that the protocols generated by this method have good performance even outside the usual regime of validity, at least for a harmonic potential with time-dependent stiffness. A summary of the results is shown in Fig. 1.

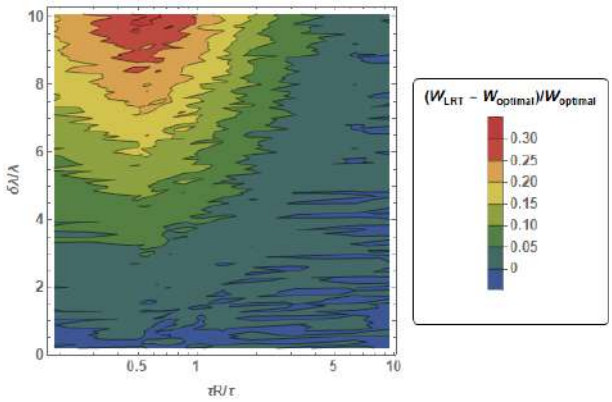

Figura 1: *The relative difference between the LRT optimal protocol and the exact solution as a function of the relative changes of control parameter and process time. It is noteworthy that the LRT protocol performs well even outside its regime of validity (for week variations: at the bottom of the graph).*

Acknowledgment

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On the trajectories of the Quasi-stationary states in the d-HMF model

R. Moreira[1*], J. A. Navarro(a)[2,†], B. Atenas[2,◇], S. Curilef[2]

[1] *Universidad Mayor de San Simón, Av. Ballivián esquina Reza 591, Cochabamba, Bolivia*
[2] *Universdad Católica del Norte, Av. Angamos 0610, Antofagasta, Chile*

* renemoreira.c@umss.edu, †juanpablo.navarro@alumnos.ucn.cl ◇batenas@ucn.cl

Introduction

In this work, we discuss some properties of a possible variation of the Ising model; this is the d-HMF model[1], which includes long-range interactions, which are of interest in atomic and astronomical scales. Typical consequences of having interacting particles in systems are lack of additivity and extensivity in the energy, and the sensitivity from the initial conditions[1-3].

The problem and results

The simulation is performed in the microcanonical ensemble by molecular dynamics. In the transition towards equilibrium the system presents a negative specific heat region below the critical temperature. The resulting instability is highly relevant because of its substantial impact on experimental and theoretical features. As generally acknowledged, several particular properties related to the nonequilibrium behaviour of systems with interacting particles, such as the relaxation to thermal equilibrium in the N-particle d-HMF model, have recently been debated. The relaxation time has been shown to belong and to increase with the number of particles. This system evolves very slowly, on time scales diverging with N through so-called "quasi-stationary states" (QSS) towards the Boltzmann–Gibbs equilibrium. The d-HMF model was recently introduced by

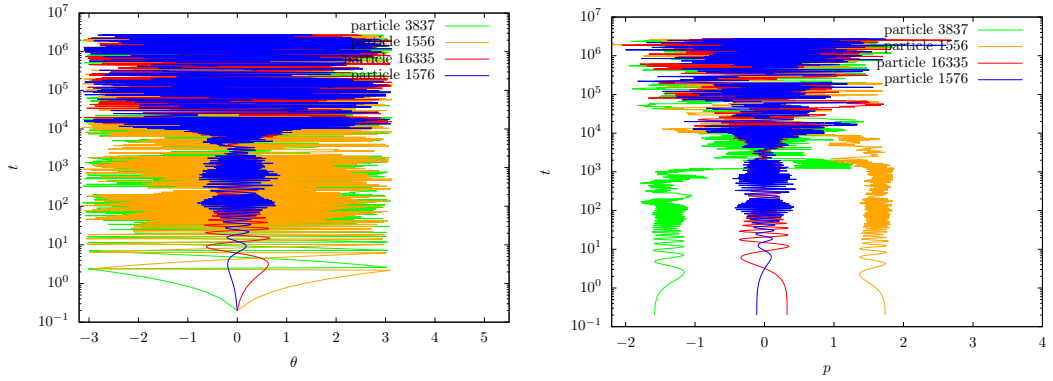

Figura 1: *In the left panel, the evolution of some trajectories in QSS regimes is depicted. In the right panel, the momentum evolution for the same particles is shown.*

Atenas and Curilef [1], and it has been previously studied in analytical and numerical[1-4] ways. In such a description, two QSS has been reported. They are described according to the behaviour of thermal properties. In this contribution, we attempt to add the perspective given from the visual inspection of trajectories of the individual particles.

Acknowledgment

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Experimental verification of optimal protocols with optically trapped colloidal particles

Thalyta Tavares Martins[1*], Lucas Prado Kamizaki[1,†], Sérgio Ricardo Muniz[1]
[1] *Instituto de Física de São Carlos, Universidade de São Paulo (IFSC-USP)*
[*] thalyta@usp.br, [†] lucas.kamizaki@usp.br

At the macroscopic level, processes involving the exchanges of energy, work, and heat are well described by the laws of Thermodynamics. However, as the energy scale decreases, fluctuations become increasingly relevant and play an essential role in the system dynamics. In this scenario, we use optical tweezers to study colloidal particles in out-of-equilibrium processes [1]. In particular, we shall present preliminary results on ongoing experiments exploring the optimization of the thermodynamic work with such a system [2].

Acknowledgment
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Statistical approaches to the problem of homogeneous melting of solids in the microcanonical ensemble

Vivianne Olguín-Arias[1*], Sergio Davis[2], Gonzalo Gutiérrez[1]

[1] *Grupo de Nanomateriales, Departamento de Física, Facultad de Ciencias, Universidad de Chile, Casilla 653, Santiago, Chile.*

[2] *Research Center on the Intersection in Plasma Physics, Matter and Complexity, P²MC, Comisión Chilena de Energía Nuclear, Casilla 188-D, Santiago, Chile.*

[*] vivianne.olguin.a@gmail.com

Introduction

Melting is a common phenomenon in our daily life, and although it is understood in thermodynamic (macroscopic) terms, the transition itself has eluded a complete description from the point of view of microscopic dynamics. While there are studies of metastable states in classical spin Hamiltonians [1, 2], cellular automata [3], glassy systems [4] and other models [5, 6], the statistical mechanical description of the microcanonical superheated solid state, a metastable state ocurring just before the onset of melting, is lacking.

Our work is oriented to the study of the melting process of superheated solids, which is believed to be caused by thermal vacancies in the crystal or by the occupation of interstitial sites [7]. When the crystal reaches a critical temperature T_{LS} above the melting point T_m, it becomes unstable and a collective self-diffusion process is triggered. These studies are often observed in a microcanonical environment, revealing long-range correlations due to collective effects, and from theoretical models using random walks over periodic lattices [8].

Our results suggest that the cooperative motion made possible by the presence of vacancy-interstitial pairs (Frenkel pairs) [9, 10] above the melting temperature induces long-range effective interatomic forces even beyond the neighboring fourth layer [11]. From microcanonical simulations it is also known that an ideal crystal needs a random waiting time until the solid phase collapses [12, 13]. Our results also point towards a description of these waiting times using a model in which there is a positive, unspecified quantity X that accumulates from zero in incremental steps, until X exceeds a threshold value X^* that triggers collapse [14].

Acknowledgments

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Sandpiles in Networks with Variable Topology

Víctor Muñoz[1*], Alejandro Zamorano[1,†]

[1] *Departamento de Física, Facultad de Ciencias, Universidad de Chile, Casilla 653, Santiago, Chile.*

[*] vmunoz@macul.ciencias.uchile.cl, [†] alejandro.zamorano@ug.uchile.cl

Introducción

Normally, sandpile models consider a grid of cells [1], where charge is redistributed to neighboring cells. Several studies [2, 3] have considered the generalized case of sandpiles in a complex network grid, where avalanches redistribute the load on the nodes connections. However, an arbitrary underlying network may have isolated nodes, which accumulate grains, or the avalanches may follow a network loop, which prevents energy release. In this work, we study the transition from a 1-D sandpile model to a more general model, without these issues, and studying the resulting sandpile statistics. These studies could be useful in contexts where dissipative processes along preferential paths are relevant, such as energy release during magnetic reconnection events. [4]

Methodology

We start with the usual BTW model, represented by a directed complex network. Avalanches proceed from node i to node $i+1$, following the direction of the network edge between them. Then, we randomly reconnect nodes in such a way that the energy can always be released from the grid, and there are no accumulation points. This is done following two rules: (a) an edge is modified only once, and (b) if the old edge goes from node i to $i+1$, the new edge goes from node i to a node $j > i+1$. This ensures that the grid is never broken.

The transition from the linear chain to the fully reconnected case is smooth, with a transition at about half the maximum number of possible reconnections. Figure 1 shows the Gini coefficient of the released energy distribution, averaged over all 15 random networks sequences, as reconnection number increases in a grid of 50 nodes. By changing the discharge paths, reconnections increase the phase space of possible energy releases, which makes its distribution less "unequal".

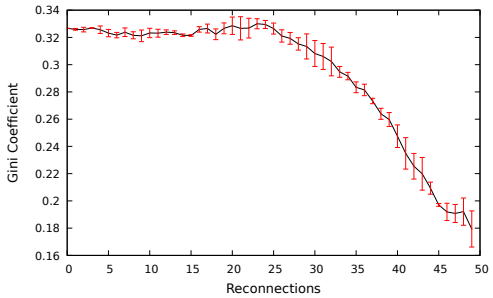

Figura 1: *Gini coefficient for the energy release distribution, with load $Q = 1$ and threshold $U = 2$*

Agradecimientos

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Statistical models to predict key performance indicators of the teaching-learning processes

Boris Atenas†, Luisberis Velazquez*
Universidad Católica del Norte, Antofagasta
†batenas@ucn.cl, *lvelazquez@ucn.cl

Introduction

The academic environment is *a complex adaptive system* formed by different agents (e.g.: students, professors, administrators, etc.) that interact among them with an restricted knowledge about each other. These agents establish stochastic patterns in their activities that follow similar behaviors than the ones observed in financial markets. In the same way that individuals in economic relations face trade-offs because resources are limited [1], the agents in an academic institution must decide how to allocate their time disposal to attempt different teaching-learning activities according to their own capacities. The study of these social phenomena can be addressed using the tools employed in *econophysics* [2]. Keystones to guarantee the quality of the teaching-learning processes are (i) the quantitative estimations of the number of academic credits (European ECTS or Chilean SCT), as well as (ii) to understand the existing relationships among different key performance indicators (KPIs). In this research, we start preliminary attempts to capture the empirical behaviors observed in the practice [3] throughout simple stochastic models inspired on physics.

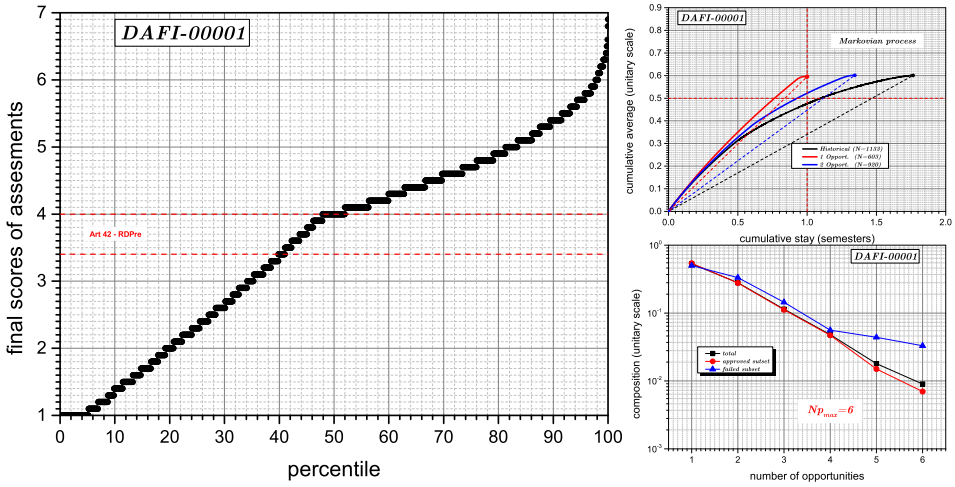

Figura 1: *Results of simulations reproducing some key statistical dependencies [4]: the historical cumulative distribution of final scores (main panel), the learning curves (upper panel) and the evolution of approved/failed subsets along opportunities (bottom panel).*

Acknowledgment

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II International Workshop on Statistical Physics
Antofagasta, Chile, December 01-03, 2021 Topic(s): **Complex Systems**

Public disorder and transport networks in the Latin American context

Carlos Cartes[1,*] and Toby P. Davies[2]

[1] *Complex Systems Group, Facultad de Ingeniería y Ciencias Aplicadas. Universidad de los Andes, Chile.*
[2] *Department of Security and Crime Science, University College London, 35 Tavistock Square, London WC1H 9EZ, UK.*

[*] carlos.cartes@miuandes.cl,

Introduction

We propose an extension of the Davies et al. model [1], used to describe the London riots of 2011. This addition allows us to consider long travel distances in a city for potential rioting population. This is achieved by introducing public transport networks, which modifies the perceived travel distance between the population and likely targets. Using this more general formulation, we applied the model to the typical Griffin and Ford pattern for population distribution to describe the general features of most large Latin American cities. The possibility of long-range traveling by part of the general population has, for an immediate consequence, the existence of isolated spots more prone to suffer from rioting activity, as they are easier to reach than the rest of the city. These areas finally made it easier to control the eventual disorder by part of police forces. The reason for this outcome is that transport networks turn riots into highly localized and intense events. They are attracting a large police contingent, which will later extinguish the remaining disorder activity on the rest of the city. Therefore, working transport networks in a city effectively reduces the number of police force contingent required to control public disorder. This result, we must remark, is valid only if the model requisites for order forces are satisfied: extra police contingent can be added swiftly as required, and these forces can move around the city with total freedom.

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University student retention analysis using quantitative tools data-based

Catalina Araya[1*], Diego González(a)[1,2,†], Sergio Curilef[1◇]
[1] *Universdad Católica del Norte, Antofagasta, Av. Angamos 0610, Chile*
[2] *Banco Itau-Corpbanca, Santiago, Chile*
[*] catalina.araya06@alumnos.ucn.cl, [†] diego.gonzalez@ucn.cl [◇] scurilef@ucn.cl

Introduction

The digital revolution, artificial intelligence and other developments and innovations have made the basic sciences, such as physics, highly relevant due to their fundamental nature[1]. The increase in calculation capacity and the massive generation of information and analysis have brought to light the full potential of statistical physics in understanding the fundamentals of data-based decision making. Thus, research associated with data science, machine learning and its applications in task optimization, along with physics and basic sciences, converge.

Decisions made by academic institutions related to helping students in the teaching-learning process can be made statistical models-based and data analysis tools, exposing the need for sufficient theoretical foundations and objective evaluations of the limitations of such models[1].

In addition, the microscopic dynamics of physical systems are reversible in time, but the macroscopic dynamics do not share this symmetry. For example, you can guess whether a movie is played correctly or in reverse order. This asymmetry in the flow of events is explained through the second law of thermodynamics. Statistical physics has extended the understanding of this problem to the microscopic regime, where fluctuations make it difficult to discern the direction of the order of events[2].

The problem and results

Here, we develop a quantitative-predictive model for UCN student retention, based on data from the Bachelor of Physics program with a major in astronomy and the careers of the Faculty of Sciences.

With the use of statistical physics and linear algebra tools, it is evaluated how relevant internal factors intervene, such as grades per subject, and external factors, such as origin, socio-economic and cultural level and other data[3] that the UCN has.

To discover the data-incidence in academic performance, we evaluate the pertinence of the logistic regression technique. We work with data from one year to predict the next year and then improve predictability by using intervals of more years to predict what is to come.

Our central hypothesis is that it is better to avoid student desertion than find new students for the UCN system. This hypothesis can be ethic-based as teaching practice-based. Then, for a student not to drop into a desertion state, we must deeply understand the previous and present features and events that determine university life. We generate a tool of early alert to evaluate appropriate options of focussing the effort on a particular student according to its necessity.

The results hope to give relevant information according to the available data on the probability that each student will drop out or not. Therefore, we expect to contribute to the policy of the UCN related to making precise decisions to improve the teaching-learning process favouring the academic success of UCN students.

Acknowledgment

We would like to thank the DGPRE No. 109/2021 "Modelo predictivo de retención-deserción estudiantil basado en herramientas teóoricas cuantitativas" for partial financial support, and the "Núcleo de Investigación No 2-Sistemas Complejos en Ciencia e Ingeniería-UCN-VRIDT 042/2020"for the scientific support.

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Complexity analysis of GOY shell model via cumulative entropy

Víctor Fernández[1†], Víctor Muñoz[1‡]

[1] *Departamento de Física, Facultad de Ciencias, Universidad de Chile, Las Palmeras 3425, Santiago.*
[†] victor.fernandez.g@ug.uchile.cl, [‡] vmunoz@macul.ciencias.uchile.cl

Introduction

Di Crescenzo *et al.* (2009) introduced the concept of cumulative entropy, which, unlike Shannon entropy, is based on the cumulative probability distribution (CDF). We use this method to calculate the cumulative entropy associated to Gledzer-Yamada-Ohkitani MHD shell model simulations. Development is currently underway to implement complexity-entropy maps based on the CDF of a time series, which would avoid the problems associated to binning involved in the calculation of the Shannon entropy.

Development

Shannon entropy of a probability distribution can be estimated directly by knowing the probabilities p_i of each state i. In the case of real time series, p_i can be derived from a binning of the possible values, a strategy that can give a bad estimate of S if the number of intervals is not properly chosen. To overcome this issue, Di Crescenzo *et al.* proposed an entropy based on the cumulative probability distribution (CDF), defined as

$$\mathrm{CE} = -\int_0^\infty F(x) \ln F(x)\, dx \ , \tag{1}$$

where $F(x)$ is the cumulative distribution function of a non-negative random variable X. Recently [3], the fractality of a magnetohydrodynamic turbulance shell model has been studied. In particular, the GOY-type shell model described by the evolution equations for the velocity $u_n(t)$ and magnetic field $b_n(t)$ fluctuations

$$\begin{aligned}
\dot{u}_n &= -\nu k_n^2 u_n + i k_n \left\{ (u_{n+1}u_{n+2} - b_{n+1}b_{n+2}) - \tfrac{1}{4}(u_{n-1}u_{n+1} - b_{n-1}b_{n+1}) - \tfrac{1}{8}(u_{n-2}u_{n-1} - b_{n-2}b_{n-1}) \right\}^* + f_n, \\
\dot{b}_n &= -\eta k_n^2 b_n + i k_n \tfrac{1}{6} \left\{ (u_{n+1}b_{n+2} - b_{n+1}u_{n+2}) - (u_{n-1}b_{n+1} - b_{n-1}u_{n+1}) - (u_{n-2}b_{n-1} - b_{n-2}u_{n-1}) \right\}^* + g_n.
\end{aligned} \tag{2}$$

corresponding to the eddy's scale of length $l \sim k_n^{-1}$. Motivated by the present study, the cumulative entropy of time series associated to the GOY shell model is analyzed, and its dependence on the level of intermittency (as measured by its fractal dimension) is studied.

Acknowledgement

We thank the support of ANID, Chile, through FONDECYT Grant No. 1201967 (VM).

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Community structure of Earth's magnetic field measurements.

Sebastián de la Maza[1,*], Víctor Muñoz[1,†]
[1] *Departamento de Física, Facultad de Ciencias, Universidad de Chile.*
[*] sebastian.delamaza@ug.uchile.cl, [†] vmunoz@macul.ciencias.uchile.cl

Introduction

The Earth's magnetic field has dependence both in the time and spatial domains. Also, due to the underlying physical processes involved, the change of the magnetic field at a given point or at a given instant may induce variations at other points and/or subsequent times. We propose to study this complex dynamics of spatiotemporal correlations by means of tools derived from graph theory and complex networks, which have shown to be useful to describe the behavior of various systems of geophysical interest [1, 2, 3]. In particular, we intend to study the evolution of magnetic field measurements on the Earth's surface along the 23rd solar cycle. Based on records by 59 magnetometers during the 23rd solar cycle (taken from the World Data Center for Geomagnetism, Kyoto, http://wdc.kugi.kyoto-u.ac.jp/hyplt/index.html), we define a complex network where nodes are the magnetometers, and their connection is determined by the correlation between their respective magnetic field time series. Thus, the structure of the complex network is expected to be a representation of the spatiotemporal patterns of the Earth's magnetic field.

Methodology

The network is defined by two similarity methods between time series, namely, the Pearson correlation [4, 5] and event synchronization [6]. Complex networks are built for each year from 1996 to 2008, covering the full 23rd solar cycle. Then, the community structure of each network is analyzed, and some of its basic features are analyzed along the cycle: the number of communities and the average area covered by the communities.

We find that the community structure, in effect, has information on solar activity. For instance, we find that the average area of the communities decreases during solar maximum, and is maximum at the start and end of the solar cycle. This suggests that the community structure of the network may reveal the changes in correlation length in the magnetic field structure as solar activity evolves. But we also show that results strongly depend on the choice of similarity methods, and the thresholds involved.

Acknowledgments

V. M. acknowledges support of ANID through Fondecyt Grant 1201967.

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Complex Networks Analysis of Solar Magnetograms along the 23rd Solar Cycle

Víctor Muñoz[1*], Eduardo Flández[1,†]

[1] *Departamento de Física, Facultad de Ciencias, Universidad de Chile, Chile*

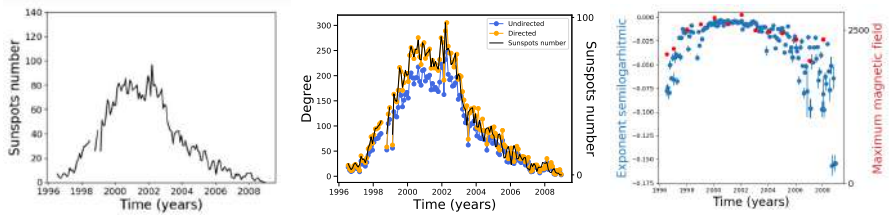
[*] vmunoz@macul.ciencias.uchile.cl, †eduardo.flandez@ug.uchile.cl

Introducción

We build monthly complex networks based on solar magnetograms for the 23rd solar cycle. [4] We use binarization, noise filtering, and image recognition algorithms to determine the sunspots centroids. Results for sunspots number are consistent with previous works. [6] Then, monthly complex networks are built using the temporal sequence of the sunspots coordinates, and various metrics are calculated. Some of them correlate, others anticorrelate, and others do not correlate at all with solar activity. Our results suggest that complex networks for solar magnetograms, built as described here, contain information on solar activity. [2] The usefulness of this method for predictions about the next solar maximum occur is currently being investigated.

Desarrollo

We first plot the number of sunspots found as a function of time. This is shown in Fig. 1(a), and is consistent with previous works. [1,5,6] We now calculate various metrics for the monthly networks, along the 23rd solar cycle. Figure 1(b) shows the average degree, both for the directed networks, and their associated undirected networks (removing directions of the edges, and self-loops). The average degree closely follows variations of solar activity. We also calculate the degree distribution for each network, and its decay exponent. Figure 1(c) shows the evolution of this exponent if an exponential behavior is assumed for the distribution tail. It also shows the maximum magnetic field registered at each month along the solar cycle. This confirms that complex networks described in this work contain physical information on solar activity. In general, we have found that metrics that correlate, and others that anticorrelate with solar activity.



(a) (b) (c)

Figura 1: *(a) Sunspots number for the 23rd solar cycle. (b) Average degree, for directed and undirected networks. (c) (Blue) Decay exponent for degree distribution. (Red) Maximum magnetic field on the solar surface.*

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Comparing the acquisition of concepts in newtonian mechanics for engineering students in different levels courses

F. Escalante[1*], F. Cuevas[1†]
[1] *Universidad Católica del Norte, Chile*
[*] fescalante@ucn.cl, [†] fcuevas@ucn.cl

Introduction

General physics plays an important role in the formation of Engineering students. Chilean engineer students in higher education have at least one course related to Newtonian mechanics, and usually, in their first year they have an introductory physics course, where converge students with a different physics background. Several authors show the freshman performance in Newtonian mechanics [1-8]. In order to measure and quantify the performance in Newtonian mechanics, The Force Concept Inventory (FCI) [9] is used. This test, in its original version, is composed of 29 multiple choice questions, split into six items: kinematics, first, second and third Newton laws, superposition principle, and kinds of force. Each choice of this test is related to one or more misconceptions probed by the inventory. The administration of this test has been used as a pre and post-instruction to measure gain learning [10,11] as well as to measure knowledge structures in physics and problem-solving skills [12,13], that although it is a skill that students must develop, it is also the understanding of concepts. The question that motivates this study is: A student that takes different courses related to Newtonian mechanics along with their curriculum, does it achieve an improvement in the acquisition of concepts in Newtonian mechanics? Some other authors suggest that learning is easily forgotten over time [14]. In this study, we compare the acquisition of concepts that engineering students ' of this study have in their physics courses related to Newtonian mechanics, along with their curriculum, considering that the students have more than one course related to this subject.

The problem and results

The misconceptions in newtonian mechanics in freshmen it is been widely studied, however, there is no literature concerning advanced undergrad students in physics courses. In order to study this gap, we administered the FCI test in three courses, located in different semesters, whose main subject is classical mechanics. The courses are Physics 1, belonging to the first semester, Physics 2, belonging to the second semester, and Physics 3, belonging to the fourth or fifth semester, according to the career. A total of 600 students answered completely the test, of which, 185 are from Physics 1, 240 from Physics 2, and 175 from Physics 3 courses. According to Savinainen et al. [14], an FCI score equal to 60 % (equivalent to 17.4 points in the 29 questions version) is the " entry threshold " to Newtonian physics, and below this value, the student's comprehension of Newtonian physics is not enough, which is the case of the students of this study, as we can see from figure 1.

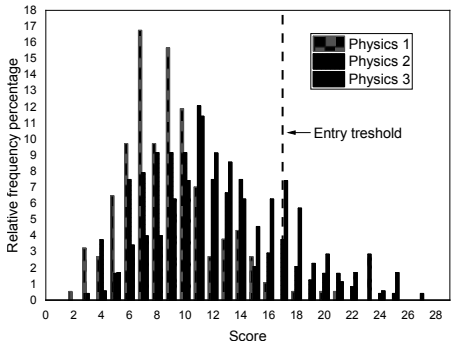

Figura 1: *Figure 1. The histogram shows the percentage of answers normalized to the numbers of students in each course.*

Considering the low performance in the three courses, our goal is to study in deep the specific questions of the test, where it concentrates on the low results. In order to achieve this goal, we use concentration analysis [15], which allows us to extract information about the answers 'distributions in the multiple-choice test. We found that both students in the advanced course as the freshmen presents confusion in velocity and acceleration concepts, which leads to misunderstanding of second Newton law. Besides, we observe an aristotelic conception of force i.e it must be acting a force on the body to maintain its motion, otherwise, there is no motion.

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II International Workshop on Statistical Physics
Antofagasta, Chile, December 01-03, 2021 Topic(s): **Complex Systems**

Quantitative measures of word distribution in Chilean annual national presidential rendition: last 30 years case

Francisco Calderón[1] and Sergio Curilef[2]

[1,2] *Departamento de Física, Universidad Católica del Norte, Av. Angamos 0610, Antofagasta, Chile.*
[1]fcalderon@ucn.cl, [2]scurilef@ucn.cl

Abstract

Complexity appears naturally in many fields, and ranges from natural science to social science and beyond. Understanding the importance of its analysis is ubiquitous to every field and development of modern techniques has made it crucial to understand both its reach and applications in science and society. Take for instance, written language, which is the representation of spoken language or gestural representation by means of a symbolic or text system. We may observe repetitive patterns in phrase or sentence formation, but to take advantage of this intrinsic information we are required to set a higher goal into finding correct ways to create and deliver information as well as understanding. Therefore, syntax and correctness of sentences have large consequences in transmitting the message properly and adequately. The content and meaningful information is remarkably hidden within the pattern construct; simply put, in terms of a wide range of word combinations. It was first observed by Zipf [1] that frequency of repetition should follow a power law, and consequently discovering a path into the inherent meaning which may have more avenues into analyzing written language and its complexity. Here, we have constructed a simple discrimination scheme based on quantitative measures that arise naturally in most written pieces [2] and from it we calculated a frequency table from political speeches using Chilean annual national presidential rendition from the past 30 years.

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Elliptical Chemoreceptors: The key to an Effective Absorption

Jason Peña[1,*], Leonardo Dagdug[1,2,†]

[1] *Metropolitan Autonomous University at Iztapalapa, CDMX, México*
[2] *Shriver National Institute of Child Health and Human Developmen, Bethesda, MD 20892, U.S*

[*] jason.penamunoz@gmail.com, [†] dll@xanum.uam.mx

Introduction

In the research, we focused on some models of chemoreception, from the perfect spherical absorbent, the use of Weber's disk, and the development of Berg and Purcell approach, to Zwanzig and Szabo generalized solution where interference effect and partially absorbing receptors are considered. Afterwards, we extrapolate Dudko's solution using a dimensional comparison for rate constant diffusion to receptors of arbitrary shape on a spherical cell, and contrast the absorption effectiveness on circular and elliptical chemoreceptors, using as reference the perfect spherical absorbent. During this process we looked to an important property related with the structure of chemoreceptors, their geometry. Is there a preferential setting that allows more particles to be absorbed in an ever smaller area? We found that the elliptical geometry offers a plausible model in cellular anatomy, a result that could explain the structure variation on chemoreceptors and the observed physiological changes on cells.

About the nature of cells

It is well known that cells are a fundamental part in the existence of life, a little organic machine capable of making big things, thanks to which we can breathe, move and reproduce. Not mentioning, that cells are responsible of the embryogenesis process. In addition to the existentiality of cells it is necessary that they work correctly, and most of the times, that depends on the cellular intercommunication, a mechanism that can be reduced into three steps; the reception, transduction and response. By the scope of the research we concentrate on the first step, the reception of signaling molecules or ligands. These particles carry useful information to the cells causing them to migrate, reproduce, or even die. A schematic representation of the cellular intercommunication is shown in Fig. 1.

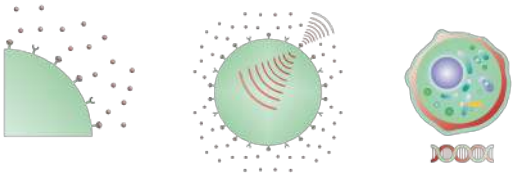

Figura 1: *Schematic representation of the intercellular information exchange sequence. From left to right, receptor-ligand interaction, transduction (amplification of molecular signals) and specific physiological response, DNA transcription in the cell nucleus are represented.*

Guided by chemical reactions, cells are a complex system that create and dismantle structures, besides making perfect copies of themselves. This biochemical network could go wrong when tiny mistakes add up until the machinery gets corrupted, something that could lead into biological disorders, like cancer. Intercellular communication is as important as the existence of cells, and the interaction receptor-ligand is the very first part of this process, a phenomenon that can be modeled with diffusion equations.

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Applications of kinetic theory to bacterial suspensions

Rodrigo Soto
Departamento de Física, FCFM, Universidad de Chile, Santiago, Chile.
rsoto@dfi.uchile.cl

Abstract

The kinetic theory of gases, in which one writes a dynamical equation for the distribution function $f(\vec{r},\vec{v},t)$ giving the average number of atoms with a given position and velocity, has been extraordinary successful. Providing only the interatomic cross section, it is possible to solve the Boltzmann kinetic equation to derive the transport laws and the values of the transport coefficients, for example, the viscosity. Similarly, for metals, the semiclassical Lorentz kinetic equation allows to obtain the Ohm's law and the value of the resistivity in terms of the quantum cross section of electrons with phonons and defects. Bacterial suspensions has become a prototype of matter out of equilibrium, where energy is supplied and dissipated at the particle scale. Indeed, each swimmer moves in the fluid balancing the thrust provided by its flagella with the viscous drag. In the past years, a systematic effort has resulted in the construction of kinetic equations to describe the motion of bacterial suspensions. Here, the object of study is the distribution function $f(\vec{r},\hat{n},t)$, giving the number of bacteria at position $\vec{r}$, swimming along the director $\hat{n}$ at time t. These kinetic equations have streaming terms, associated to the free swim at speed $\vec{V}=V\hat{n}$, Boltzmann-like terms for the mutual alignment that result from collisions, and a Lorentz-like term for the tumbling process, in which a new director $\hat{n}'$ is chosen at random. Also, rotational diffusion changes continuously the director, process that is modeled with a Fokker-Planck term in the kinetic equation. Finally, bacteria also interact hydrodynamically, through the flows generated by their flagella. These interactions are long-ranged, allowing to treat them in a mean field approach with a Vlasov-like term in the kinetic equation. Using standard and ad-hoc methods, we have analyzed the collective properties of bacterial suspensions using a kinetic approach. In particular, we have derived the time it takes to reach the diffusive regime and computed the associated diffusion coefficient, obtained the active contribution to the stress, analyzed the spatio-temporal chemotactic response, and showed that for E. coli, the tumbling rate is not constant, but rather presents large fluctuations, allowing swimmers to easily contaminate narrow channels.

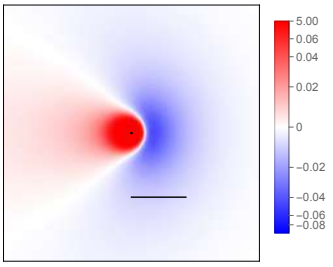

Figura 1: *Bacterial wake that results from the chemotactic response to a traveling wave of nutrients concentrated in the black dot.*

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Analysis of Oscillatory Time series using the Visibility Graph method

N. Elizabeth Garcés H.[1,*], Víctor Muñoz
[1] *Departamento de Físicia, Facultad de Ciencias, Universidad de Chile*
[*] nayade.gh@gmail.com

Introduction

Complex networks are a useful tool to study various nonlinear systems, and their application to the analysis of time series is interesting, since time series are the basic input that we have to study nature. The (horizontal) visibility graph (VG/HVG) method [1, 2] is one proposal to build complex networks from time series, and recently [3], we have used it to study the light curves of pulsating variable stars (Cepheids, δ Scuti, and RR Lyrae stars), investigating the method's ability to discriminate between them.

However, datasets may be of different lengths. Second, although all studied stars have oscillatory light curves, the complex star dynamics introduces various noise levels. And third, light curves are interrupted by observational gaps, due to geometrical or instrumental constraints.

This may affect the results of a VG analysis, and its discriminating capabilities. Thus, in this work, we use the (horizontal) visibility graph method to build graphs from artificial time series. Inspired by the issues mentioned above, we focus on sinusoidal time series of various frequencies, and studying the robustness of various complex networks metrics under modification of the time series. First, we study the effect of the length of the time series. Then, we consider sine series polluted with two types of noise: gaussian and chaotic. And finally we study the effect of gaps in the time series.

Results

For the various time series, we studied its degree distribution, clustering coefficient, and transitivity coefficient. We find that metrics depend very weakly on time series length. Regarding noise, we also observed that metrics are not too sensitive to it, except for the degree distribution, where a small noise level is enough to modify the degree distribution (Fig. 1). Finally, we find that the (H)VG method is hardly affected by gaps, making it an interesting technique to study time series where observational gaps are unavoidable, as is the case of stars observations. This is consistent with the results already found in Ref. [3].

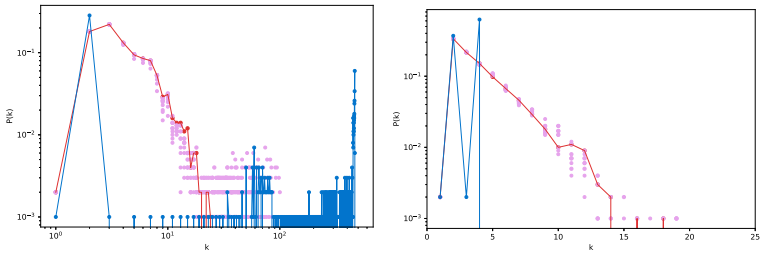

Figura 1: *Degree distribution for pure sine series (blue), polluted with different amounts of gaussian noise (pink) and pure gaussian noise (red).*

Acknowledgments

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LMC COMPLEXITY ANALYSIS FOR WEIBULL DISTRIBUTION

Oyarce.R[1*], Pennini.F[2,†]
[1] *Universidad Católica del Norte, Antofagasta, Chile*
[2] *Universidad Católica del Norte, Antofagasta, Chile*
[*] rodrigo.oyarce@alumnos.ucn.cl, [†] fpennini@ucn.cl

Summary
The present work studies the parameters of the Weibull distribution model, showing the usefulness of the statistical complexity measure provided by Ricardo López-Ruiz, Héctor Mancini and Xavier Calbet in areas related to the survival of the parametric Weibull distribution, With which it is necessary to obtain an expression of the survival probability, in this way to obtain different analytical expressions that will be analyzed and the analytical scope of the resulting expressions will be observed. The relationship between the Weibull distribution and the irrigation function is effectively described, with which the analytical probability function of the model is constructed. As a result, the expressions for Shannon's entropy, Desequilibrium and LMC Complexity are obtained, where they are analyzed with respect to the scale parameters t_0 and in the form n. It is concluded that the forms of the complexity are proportional to those presented by López-Ruiz et.al, in addition to identifying a possible transition of the system itself.

Introduction
For several decades different studies have tried to simulate real processes of living organisms, the difficulties to approach this type of systems become evident from the point of view of physics, which is that living organisms are thermodynamically open systems and their organization is complex. due to the network of molecular interactions, the combination of these characteristics allows these complex systems to show emergent or self-organized properties [1].
For this reason, it is interesting to analyze the unknown and widely variable behavior, which leads us to ask how the microscopic particularities could be studied in a macroscopic way? For this, Statistical Mechanics has sought answers. In particular, Ricardo López-Ruiz, Héctor Mancini and Xavier Calbet contributed a revolutionary statistical measure, called Statistical Complexity [2]. This has been at the forefront in different studies and models, both physical and economy.
In relation to what has been described above, Statistical Complexity could provide structural details in the study of the Weibull parametric survival function, managing to study the analytical scope of the system.

Results

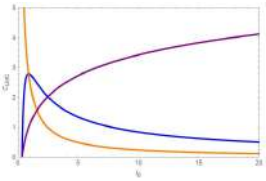

Figure 1: *Statistical measures, Complexity, Entropy and Desequilibrium*

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Nonlinear diffusion-reaction equation: Solutions and applications

Sergio Curilef

Departamento de Física, Universidad Católica del Norte, Av. Angamos 0610, Antofagasta, Chile
*scurilef@ucn.cl

Introduction

Partial differential equations, their solutions, and their applications have been paid attention to in physical and mathematical phenomena involved in modeling the behavior of several complex systems. For instance, they are serious candidates for analyzing the growth and spread of special populations. These nonlinear evolution equations are applied in fields such as ecology and archaeology; mainly, nonlinear diffusion terms were used in pioneering work in the field of astrobiology [1]. In addition, these equations have been turned into a widely applicable tool that has provided valuable insights in statistical physics, nonlinear optics, quantitative biology, and finance, among other areas. A family of these equations often permits analytical solutions to exhibit a maximum q-entropy form. Under proper simple constraints, such solutions play a prominent role in novel applications of evolution equations involving nonlinear, power-law diffusion.

Basic equation and solutions

We take a function $W = W(x,t)$ which represents the evolution of a population distributed in a space x and fulfills the following equation

$$\frac{\partial W}{\partial t} = \frac{\partial}{\partial x} D(W) \frac{\partial W}{\partial x} + rW - \bar{\mu} W^m,$$

where $D(W) = \kappa W^m/(m+1)$, and κ, r and $\bar{\mu}$ are the diffusion, reproduction and competition parameters, respectively. Choosing $n = 1 - m = q$, we obtain the solution

$$W = \left(A(t) - (1-q)\frac{(x-y(t))^2}{4S(t)} \right)^{\frac{1}{1-q}},$$

which produces a set of motion equations for the variables $y(t)$, $A(t)$ and $S(t)$, whose interpretation related to the distribution is: the amplitude $A(t)^{1-q}$, the width $\sqrt{A(t)S(t)}$ and the maximum $y(t)$.

Applications and results

It is well-known from laser-annealing experiments on planar surface layers of silicon and germanium using nanosecond synchrotron x-ray pulses as heat enters the sample. On the baseline of our motivation, we can put in the heat equation to get the temperature distribution W represents the temperature T taking a nonconstant heat conductivity, as $D \propto T^{-1,23}$[2]. We obtain a temperature profile that evolves more slowly than predicted by standard theories based on Fourier law.

In addition, we propose an analitycal methodology to understand the macroeconomical evolution of per capita income in countries where there are social outbreaks. Certainly, this is a problem that has political, psychological, economic, ideological, and other implicates. In our model, beyond any ideological or59subjective consideration, we consider the increase of the Gross domestic products (GDP) that competes in an extended nonlinear diffusion reaction equation with the Consumer price index (CPI)[3]. According to our results, the concavity of employee income distribution is a decisive input parameter and, in contrast to the distributions typically observed for Chile and other countries in Latin America, should ideally be non-negative.

Acknowledgment

We would like to thank to the "Núcleo de Investigación No 2-Sistemas Complejos en Ciencia e Ingeniería-UCN-VRIDT 042/2020"for the scientific support.

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How do our students follow the teaching-learning process of online courses?

L. Velazquez[†], F. A. Calderon[*], V. Bahoz-Mamani
Universidad Católica del Norte, Antofagasta
[†]lvelazquez@ucn.cl, [*]fcalderon@ucn.cl

Introduction

We present a quantitative study of some online courses developed during COVID19 sanitary emergency in Chile. Our interest is focussed on the rebuilding of teaching-learning processes (see Figure 1) considering the activity records of digital platforms (e.g.: *Zoom*, *YouTube*, *Moodle*) in order to answer the question *How do our students study?* The analysis evidences the *complex adaptive character* of the academic environment, which exhibits statistical patters fully similar to the ones found in financial markets [1,2] (e.g.: distributions of the daily time devoted to learning activities follows patterns like Pareto's or Zipf's laws). Our empirical results demonstrate both (i) the relevance of *econophysics* in the understanding of the *teaching-learning processes* and (ii) the relevant role of quantitative methods based on digital platforms to conduct experimental studies within this multidisciplinary scenario.

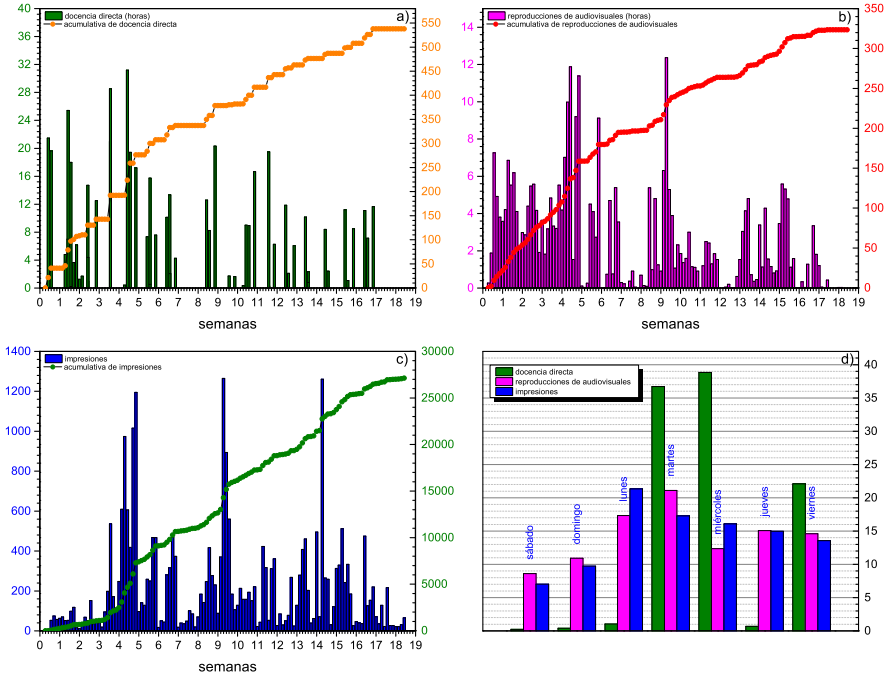

Figure 1: *Rebuilding of the daily time devoted to learning activities by a group of students that follows the online course* Newtonian Mechanics *via digital platforms.*

Acknowledgment

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Complex networks approach for studying polarization in different social groups

Verónica Subieta-Frías*, Gonzalo Marcelo Ramírez-Ávila†
Instituto de Investigaciones Físicas. Universidad Mayor de San Andrés
*vsubieta@fiumsa.edu.bo, †mravila@iumsa.edu.bo

Introduction

The study of social systems has become an important topic in complex systems that sometimes is also called Sociophysics. Several features are studied, such as consensus and polarization in social groups. The polarization of opinions related to a controversial issue in a social group of N individuals is analyzed. The study is based on a discrete-time model of opinion formation similar to the one studied in [1] and [2]. This model considers that the opinion of each individual evolves influenced by both its own opinion and the other individuals' opinion who are members of the same social group or network (friends or acquaintances), which is characterized by an adjacency matrix. The state of the opinion of each individual takes continuous values in the interval [0, 1]. The initial opinion of each individual is randomly chosen considering a Gaussian distribution with a mean value of 0.5.

The problem and results

As a starting point, we consider a social network model similar to the one proposed by [1], which is based on a discrete evolution. The network consists of N individuals (agents), each of which has a state variable.

$$S_j\left(t+1\right) = \frac{S_j\left(t\right) + \sum_i a_{ij} S_i\left(t\right)}{1 + \sum_i a_{ij}} \text{ ,with } , j = 1, 2, ..., N \ , \tag{1}$$

where $S_j(t)$ is the opinion state at time t and $S_j(t) \in [0, 1]$. The adjacency matrix a_{ij} represents the network of acquaintances and friends of the individual j.

For the study of polarization, we introduce an index similar to that used in [3] based on the number of individuals who acquire extreme opinions, i.e., intransigents [4], concerning the total number of individuals in the social group. This index is defined as:

$$R_{\%} = \left(1 - 2\left(0{,}5 - \min\left\{fp, fn\right\}\right)\right) * 100 \ , \tag{2}$$

where fp and fn are the fractions of individuals who have an extreme opinion, belong to a threshold number near to one or zero.

The results for the polarization index $R_{\%}$ were obtained for different fractions of susceptible individuals and several average degrees of free scale networks, as shown in Fig. 1.

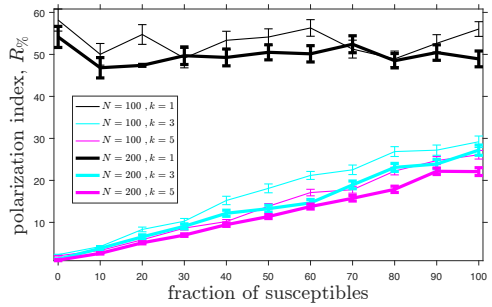

Figura 1: *Polarization index as a function of the fraction of susceptible individuals, varying the number of individuals in the network and considering the average degree of a scale-free network.*

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II International Workshop on Statistical Physics
Antofagsta, Chile, December 01-03, 2021 Topic(s): Interdisciplinary research

Proposal of Sudden Death Indicators based on Information Measures from ECG Signals

Pablo Martínez-Coq[1,*], Andrea Rey[1,†], Walter Legnani[1,‡], Osvaldo A. Rosso[2]

[1] *Centro de Procesamiento de Señales e Imágenes, Universidad Tecnológica Nacional Facultad Regional Buenos Aires, Argentina*
[2] *Instituto de Física, Universidade Federal de Alagoas, Brazil*

[*] pablomartinezcoq@gmail.com, [†] arey@frba.utn.edu.ar, [‡] walter@frba.utn.edu.ar

Introduction

Millions of people around the world die suddenly every year. Sudden death is the consequence of a final arrhythmia, which may be produced by a wide variety of different electrical and mechanical substrates. Among others, it can be mentioned an acute myocardial infarction, myocarditis, coronary diseases, cardiomyopathies, syndromes such as a repolarization of the heart after a heartbeat -known as LQTS-, Brugada and Wolf Parkinson White, or drug toxicity. However, the most important aspect related to this disease is that some people have no evident clinical predisposition. Thus, the aim of this paper is to present a novel study that may provide important clues to the pathogenesis of the sudden death syndrome (SDS).

Since the information content in the time series from an ECG signal is conveyed in the form of a probability distribution function, in such a way the Permutation Entropy proposed by Bandt and Pompe is applied to calculate it, because it has a remarkable conceptual simplicity, computational speed and robustness to noise.

In previous studies, a new approach was proposed based on the individualized computation of the embedding time delay τ for each ECG signal. Then, the normalized Shannon entropy and the statistical complexity measure defined by the Jensen-Shannon divergence were calculated from a database containing ECG records coming from people with normal sinus rhythm and other cardiac pathologies such as arrhythmias. A feature set formed by these three variables was used in a classification model applying the random forest technique that made it possible to discriminate both groups of patients.

The main goal of the present work is to formulate a model based on indicators from the analysis of the evolution of these variables along the time in 24 hour holter's ECG recordings coming from people who are supposed to die suddenly. In this way, the relevance of this study is to describe and point out potential behaviors or characteristics that can be used to make a premature detection of the SDS.

The ECG records used in this study were obtained from the MIT-PhysioNet database which is available at https://physionet.org/content/sddb/1.0.0/. The signals were windowed through a time segmentation of non-overlapping partitions of different lengths to analyze the influence of the amount of samples under consideration. The obtained results have shown a concurrent variation of the information measures used with the emergence of atypical patterns that are clearly identified in the ECG signals.

On one hand, the values of the statistical complexity measure for reported ECG with possible SDS were bigger than those of the normal sinus rhythm group, and it is vice-versa for the Shannon entropy values. The behavior of the time embedding delay parameter along the time agreed exactly with the manifestations of both, the Shannon entropy and the statistical complexity measure changes which accompanied the anomalous shapes of the ECG signals. Moreover, the values of τ obtained remained in the range reported by the available literature. In this sense, these results are coincident with the obtained from ECG with arrhythmic records. The variation of the τ values when an anomaly arises in the ECG, would indicate a temporal lack of correlation of the signal, becoming an aspect that could contribute to characterize the syndrome under study.

Fluidized State Stability study from Radioactive Particle Tracking Results

G. Salierno[1*], M. Maestri[2], J. Picabea[2], C. De Blasio[1]
M. Cassanello[2], D. Hojman[3], M-A. Cardona[3], H. Somacal[3]
[1] *FNT, Energy Technology, Abo Akademi University, Vaasa , Finland.*
[2] *LARSI, Dto. Industrias, FCEyN, Universidad de Buenos Aires, Argentina.*
[2] *LaDiR, Dto. Física Experimental, Comisión Nacional de Energía Atómica (CNEA), Buenos Aires, Argentina.*
[*] agabriel.salierno@gmail.com

Introduction

Radioactive Particle Tracking (RPT) data enabled the experimental stability analysis of freely moving particle trajectories within pilot-scale fluidization equipment, operated as fluidized beds, which are difficult to obtain otherwise [1]. From the analysis of RPT trajectory manifolds, a normalized form of Shannon entropy arises as a useful statistic to study the homogeneity, mixing, and stability of the fluidized particles.

Methodology

We study the motion of fluidized calcium alginate spheres under the influence of an upward fluid flow within a $1{,}2m$ high and $0{,}1m$ inner diameter column. The radioactive particle tracking technique is a proper methodology to study the internal dynamics of these kinds of equipment. Two modes of fluidization were considered in this work [2]:

Liquid–solid fluidized bed (LSFB), where the solid phase motion is driven with the liquid flowing upward. In this case, there is no gas flow.
Gas–liquid–solid fluidized bed (GLSFB), where the liquid and the particulate solid phase are contained in the column while the gas flows upward.

Results and discussion

Normalized Shannon entropy (Ω) time evolution resulted an excellent tool for the quantitative determination of mixing times [2] and stability [3]:

$$\Omega(t) = -\frac{\sum_{n=1}^{N} p_n(t)\log(p_n(t))}{\log(N)} \tag{1}$$

Axial profiles of mixing times in the GLSFB point to a sharp minimum at $0{,}15m$, interpreted as a hot spot of turbulence. On the other hand, the LSFB shows a smoother axial profile of mixing times. Mixing times are remarkably similar in order of magnitude, pointing out that convergence to stability is a major driver of mixing. Additionally, the Glansdorff–Prigogine stability measure [3] can accurately capture flow regime transitions of the gas–liquid–solid fluidized bed, allowing it to be used to construct reliable operative windows for fluidization equipment.

Acknowledgment

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Filling phenomena in microfluidics and front microrheology of biological fluids

Agatha Pinto-Pino[1*], Claudia Trejo-Soto[1*]

[1] *Instituto de Física, Facultad de Ciencias, Pontificia Universidad Católica de Valparaíso.*
[*]agatha.pintop@gmail.com,[*]claudia.trejo@pucv.cl

Introduction

The presented method in this work is based on the functioning of microrheometry and it is able to determine the viscosity of a fluid using the fluid-air interface dynamics driven by hydrostatic pressure, inside a rectangular microchannel. First, we worked at a constant velocity regime, where the viscosity of four newtonian fluids were characterized: water, ethylene-glycol, 50 % glycerol and blood plasma. This method was able to accurately characterize the viscosity of all fluids but the ethylene glycol, where a discrepancy of more than 20 % was found when comparing to values of other studies cited in this work. Then, we applied this method to blood samples of 45 % and 35 % hematocrit, at the same velocity regime and we were capable of calculate the viscosity of these samples and the method was able to distinguish blood concentrations with different hematocrits. Finally, this method was applied in variable velocity regime, where it was possible to characterize the viscosity of the water, at different positions of the microchannel, obtaining a good correlation with a theorical model simulation.

Results

A solution for the Stokes equation [1] is the Couette flow, which yields a relationship between the fluid's viscosity, its stress and shear rate as

$$\eta = \sigma \dot{\gamma}^{-1} \tag{1}$$

Now lets consider a coupled system consisting of a tube of radius r and length l_t connected to microchannel of width w and height b. The viscosity of a newtonian fluid flowing through the microchannel, driven by a pressure difference ΔP, can be expressed as [2]

$$\eta = \frac{\Delta P}{\left[\dfrac{8 l_t b w}{\pi r^4} + \dfrac{12 x(t)}{b^2\left(1 - 0{,}63\frac{b}{w}\right)}\right]\dot{x}(t)}. \tag{2}$$

Whilst for non newtonian fluids, we can use a power-law model, also known as the Ostwald-de Waele model given by [3]

$$\eta(\dot{\gamma}) = m\dot{\gamma}^{-1}, \tag{3}$$

where m corresponds to the value of viscosity at $\dot{\gamma} = 1$.
The mean velocity of the fluids front was obtained by recording its motion through the microchannel at a fixed height.

Acknowledgments

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Characterization of the aggregation dynamics and the viscosity of blood in inflammatory diseases

I. Hinojosa[1], A. Moreno-Martínez[1], L.F. Aguilar[2], C. Cantillana[3], F. Martínez[4], G. Varas[1], A. Hernández-Machado[5], C. Calero[5] and C.A. Trejo-Soto[1*]

[1] *Instituto de Física, Pontificia Universidad Católica de Valparaíso. Casilla 4059, Chile.*
[2] *Instituto de Química, Pontificia Universidad Católica de Valparaíso. Casilla 4059, Chile.*
[3] *Tecnología Médica, Pontificia Universidad Católica de Valparaíso. Casilla 4059, Chile.*
[4] *Escuela de Ingeniería Civil, Pontificia Universidad Católica de Valparaíso. Casilla 4059, Chile.*
[5] *Departament de Física de la Matèria Condensada, Universitat de Barcelona (UB), Spain.*

[*] claudia.trejo@pucv.cl

Introducción

Aggregation of red blood cells is the tendency of cells to form stacked structures, commonly known as rouleaux. The formation of these structures is related to macromolecules present in plasma (i.e. fibrinogen and dextran) [1]. These structures have several characteristics such as, the number of red blood cells per rouleau is variable and side to side formations are possible. Figure 1 shows an image of a large aggregate formation in a blood sample of 38 % hematocrit, where some of these characteristics are observed.

On the other hand, erythrocyte sedimentation occurs due to differences in the specific gravity between red blood cells and blood plasma, and it can be separated in 3 stages. The first stage is the aggregation of red blood cells. The second stage is the sinking of the aggregates at constant speed, due to their densities. The final stage is the settling of the aggregates at the bottom of the tube [2]. Therefore, sedimentation rate is directly related to red blood cells aggregation. Furthermore, red blood cells aggregation is a phenomenon with considerable hemodynamic and hemorheological consequences.

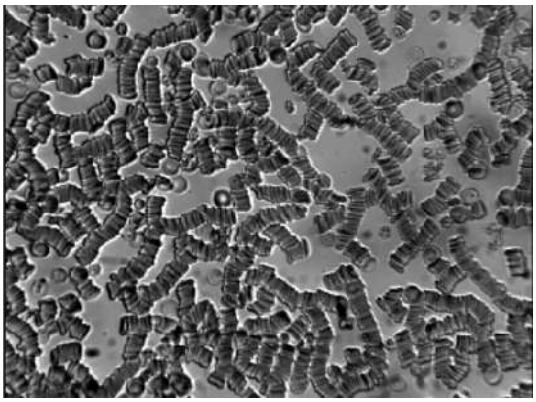

Figura 1: *The image shows red blood cells aggregates, of a blood sample at 38 % hematocrit, on a glass slide. The image was taken with a 50X magnitude ocular*

It is widely known that blood is a non-Newtonian fluid, specifically it presents a shear thinning behavior, meaning that, its viscosity decreases as stress or shear rate increase. This particular behavior of blood depends on the characteristics of RBCs: concentration, aggregation and RBC's deformability [3]. When RBCs are aggregated more stress is needed to move the fluid, but as stress increases all RBCs start to disaggregate making it easier to change the state of motion of blood. If we keep increasing the stress, cells start to deform and elongate. Therefore, if aggregation is increased then blood viscosity is increased as well, moreover, the shear thinning behavior of blood is altered [4].

The relation between red blood cells aggregation and blood viscosity with pathological conditions has been widely reported. Some related to inflammatory processes as allergic disorder and rheumatoid diseases

or other diseases such as circulatory disorders, obesity and diabetes. Rheumatic diseases are among the most common examples of clinical conditions with enhanced red blood cells aggregation due to chronic inflammation, moreover, ESR values are significant markers for clinical evaluation of these diseases [5].

We postulate that since fibrinogen concentration is increased in some inflammatory diseases, the increased concentration will affect not only the result of ERS but the dynamics of rouleaux formation and the viscosity of blood. Therefore, comparing the dynamic of RBCs aggregation with ERS results, for healthy and unhealthy blood samples, we can determine an indicator parameter of inflammation. Moreover, we attempt to determine a factor that may lead us to describe the aggregation process using self-assembly models and to propose an automatized method to analyzed aggregation that could be incorporated to perform fast clinical diagnostics.

We propose the development of an alternative method for clinical markers of inflammatory diseases, analyzing the dynamic of RBCs aggregation and the rheological parameters of blood. We attempt to reduce the analysis time and use a small amount of sample by means of microfluidics technology and image analysis techniques.

Agradecimientos

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Characterization of red blood cell aggregation

Ignacio Hinojosa[1,*]
[*]ignacio.hinojosa.c@mail.pucv.cl

Introduction

Aggregation is a characteristic of red blood cells (RBCs), which consists in the tendency of red blood cells to form organized stacked structures named rouleaux. This process is reversible such that aggregates will reform at stasis upon the removal of external forces [1].In addition, the in vivo flow dynamics and flow resistance of blood are influenced by RBCs aggregation. Abnormal increases of RBCs aggregation have been observed in several diseases associated with vascular disorders as diabetes mellitus or hypertension [2].

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Percolation Detection using Convolutional Deep Neural Networks

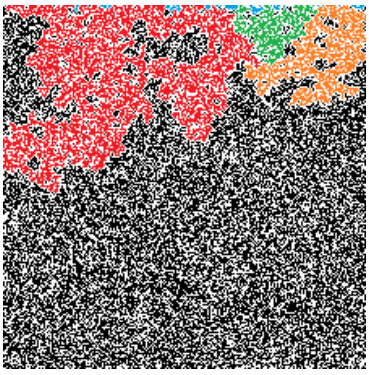
Esteban Iriarte[1*], Joaquín Peralta[1†], Claudia Loyola[1], and Sergio Davis[1,2].

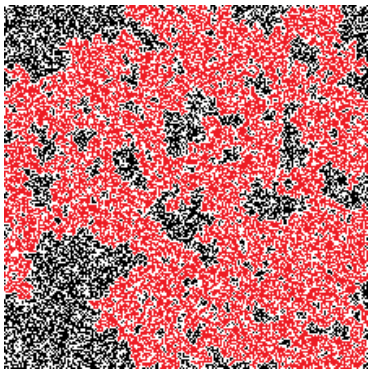
[1] *Departamento de Física, Facultad de Ciencias Exactas, Universidad Andres Bello, Santiago, Chile.*
[2] *Research Center on the Intersection in Plasma Physics, Matter and Complexity, P²MC, Comisión Chilena de Energía Nuclear, Santiago, Chile.*

[*] esteban.iriarte21@gmail.com, [†] joaquin.peralta@unab.cl

Introduction

This work describes the percolation phenomena[1] in different structures through deep neural networks[2] and previously calculated statistical data of percolation. The percolation phenomenon refers to the movement and filtering of fluids through porous materials. Despite being relatively simple and easy to calculate at small scales, the percolation process is computationally time-consuming at large scales. Significant computation is necessary to determine if a cluster percolates or not at large scales. In order to address this challenge, we propose to train deep neural networks on small systems and scale to large systems. The figure shows two different cases with a structure described by black and white pixels –black blockss the fluid and white allows it–, here the fluid moves through this structure by percolation. The figure shows that even when the structures may appear similar, percolation is present only in one case. The left case does not percolate because the fluid can not move down the structure; otherwise, the right side shows a complete percolation from top to bottom.





Using convolutional neural networks[3] to study an image, we are able to determine if some medium percolates or with what probability it can percolate. The usefulness of this method is that we can train our neural network by simulating our media and port it to new systems and potentially obtain valuable predictions. Our results could already be applicable to use cases in real life[4], such as fire spreading throughout a forest, detecting structural flaws in concrete or classification of thunderstorm patterns with just a picture.

Acknowledgment

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Inference for Unreliable Grading: The Case of Recommendation Letters

Sergio Davis[1,2*], Joaquín Peralta[2], and Claudia Loyola[2].

[1] *Research Center on the Intersection in Plasma Physics, Matter and Complexity, P²MC, Comisión Chilena de Energía Nuclear, Santiago, Chile.*

[2] *Departamento de Física, Facultad de Ciencias Exactas, Universidad Andres Bello, Santiago, Chile.*

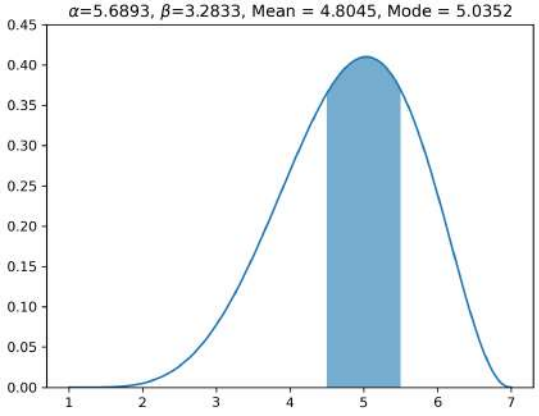
[*] sergio.davis@cchen.cl, [†] joaquin.peralta@unab.cl

Introduction

Frequently the use of a well-established criteria to determine the reliance in a assigned score is an extremely complex issue. We present in this work a procedure to found the distribution to be assigned to a score t, with $t \in [0,1]$ using Bayesian inference. To set t, we assume a resolution Δt to assign a a score of t^*, so that $t \in [t^* - \Delta t/2, t^* + \Delta t/2]$ with a confidence p. By the use of Jaynes' Maximum Entropy Principle[1] and several techniques from statistical inference[2], we propose a detailed procedure to determine the expected value $\langle t \rangle$ and its underlying distribution.
We start by assuming a Beta distribution for t and reframe the problem as the search for α and β, such that

$$\int_{t^* - \frac{\Delta t}{2}}^{t^* + \frac{\Delta t}{2}} dt \cdot t^{\alpha - 1} (1-t)^{\beta - 1} = p \cdot B(\alpha, \beta) \tag{1}$$

with the (α, β) pairs that maximize the entropy $S_t(\alpha,\beta) = \log(B(\alpha,\beta)) - (\alpha-1)\psi(\alpha) - (\beta-1)\psi(\beta) + (\alpha+\beta-2)\psi(\alpha+\beta)$, with $\psi(Z) = \frac{d}{dZ}\log\Gamma(Z)$ the digamma function. Using Lagrange multipliers we are able to determine an expected value of t, and found the (α,β) parameters of the model.



The Figure shows the results provided by numerical solution for the case of an initial t of 5 with a $\Delta t = 0.5$. Here the α and β estimations corresponds to the value of p closest to the expected probability.

Our results provide the best parameters based in a confidence p in, for instance, a recommendation letter. By developing this method we are able to narrow down and better represent our score in the letters.

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Response of cellular tissues to active stresses

Rodrigo Soto[1*], Fernanda Pérez-Verdugo[1], Jean-Francois Joanny[2,3], Germán Reig[4], Mauricio Cerda[5,6], Miguel L. Concha[5,6],

[1] *Departamento de Física, FCFM, Universidad de Chile, Santiago, Chile.*
[2] *Collège de France, 11 place Marcelin Berthelot, 75005 Paris, France.*
[3] *Institut Curie PSL University 26 rue d'Ulm 75248 Paris Cedex 05.*
[4] *Escuela de Tecnología Médica y del Centro Integrativo de Biología y Química Aplicada (CIBQA), Universidad Bernardo O'Higgins, Santiago, Chile.*
[5] *Faculty of Medicine, Universidad de Chile, PO Box 70031, Santiago, Chile.*
[6] *Biomedical Neuroscience Institute, Independencia 1027, Santiago, Chile.*
[*] rsoto@dfi.uchile.cl

Abstract

The morphological changes in the early embryo development, which finally lead to the generation of organs, is in many cases driven by constriction forces in cells. These forces, acting on single or group of cells pull neighboring cells, reshaping the whole tissue. Here, we analyze the response of epithelial tissues to changes in the active stresses in two limiting cases: a) when whole tissue (or large part of it) is subject to an homogeneous contraction and b) when only one cell contracts. For the analysis, we use the 2D vertex model of epithelia, where the vertices move such that cells approach target areas and perimeters. Active contractions are included in the model as instantaneous changes in these target values, generating active stresses. For the global activities, it is found that for a wide range of model parameter, the tissue as a whole becomes unstable in the form of soft model, where first cells elongate to later become non-convex [1]. In the case of a single active cell, the system responds quite differently depending if the activity is in the perimeter or in the area. In the first case, the active cells contract appreciably, adopting circular shapes, while in the later, the contraction is weaker and the cells elongate. This difference in geometry allowed us to apply the model to determine where the activity was present in experiments performed in the killifish *Austrolebias nigripinnis*, where isolated cells contact. The geometrical evolution is accessible by microscopy and this information alone allowed us to determine that the perimeter was the active one and to fit all the parameters of the vertex model [2].

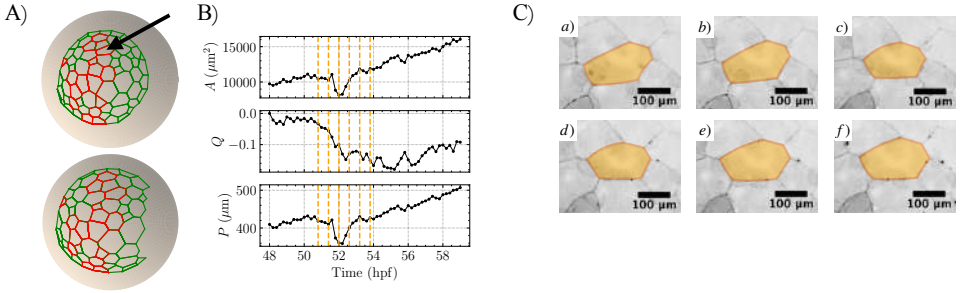

Figura 1: *A) Temporal progression of epithelial cell shape changes of Austrolebias nigripinnis from 48 to* 59 hpf*. Epithelial polygonal EVL cells are denoted in green and those with notable constriction through the timescale considered are shown in red. B) Temporal evolution of cell shape changes referring to cell area A, change of anisotropy Q and perimeter P that characterise the constriction in the cell marked by a black arrow in A). C) Representative confocal microscopy images of a single EVL cell (in orange) suffering a transient event of apical constriction. The duration of the complete image sequence of cell shape changes is 3h.*

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Non-Commutative Bayesian Expectation and its Connection to Quantum Theory

Yasmín Navarrete[1,*], Sergio Davis[2,3,†]

[1] *Instituto de Filosofía y Ciencias de la Complejidad. Los Alerces 3024, Ñuñoa, Santiago, Chile.*

[2] *Research Center on the Intersection in Plasma Physics, Matter and Complexity, P²MC, Comisión Chilena de Energía Nuclear, Santiago, Chile.*

[3] *Departamento de Física, Facultad de Ciencias Exactas, Universidad Andres Bello, Santiago, Chile.*

[*] yasmin.navarrete@gmail.com, [†] sergio.davis@cchen.cl

Introduction

Bayesian probability [1, 2] considers probabilities as degrees of plausibility that must be updated according to newly available information or evidence. In the standard application of this theory, the possible side effects that an act of measurement can have are not considered and, because of this, the results of the updating are not dependent on the ordering of two such measurements. However, the need for a Bayesian theory of non-commutative measurements arises in different contexts, from the analysis of physical experiments where the diagnostic equipment cannot be easily decoupled from the phenomenon of interest, destructive measurements in several contexts, to the understanding of the influences that a question may have on following questions in the context of social studies.

In this work we present a simple proposal for the application of the standard Bayesian probability theory to non-commutative measurements on a generic system, physical or otherwise. We show that the resulting formalism can be cast in an abstract way which is surprisingly close to quantum theory, together with a complex Hilbert space, linear operators representing measurements and an *epistemic* density operator that does not represent a physical entity but encodes a state of knowledge, in a similar manner as the so-called *Quantum Bayesianism* [3] (or QBism) interpretation of quantum mechanics.

Our results may be relevant to the understanding of some phenomena in human cognition where the mental state changes as a result of previous judgements [4], as well as shed light on the origin of quantum-like phenomena at macroscopic scales [5, 6] and even serve as a basis for the modelling of the effect of probe diagnostics in non-equilibrium experiments [7].

Acknowledgements

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II International Workshop on Statistical Physics
Antofagsta, Chile, 01-03 de diciembre de 2021 Topic(s): Dynamical Systems

Tricorn-like structures in the Lorenz-84 low-order atmospheric circulation model

C. Abraham T. Chávez[1,2,3*], Patricio Alvear Opazo[2], Sergio Curilef[2,†]
[1] *Universidad Católica "San Pablo"-Tarija, Calle Colón 734, 541030 Tarija, Bolivia.*
[2] *Universidad Católica del Norte, Avenida Angamos, 0610 Antofagasta, Chile.*
[3] *Universidad Mayor de San Simón, Cochabamba, Bolivia.*
*ctorrico@ucb.edu.bo, †scurilef@ucn.cl

In this work, we report the existence of *tricorn*-like structures of stable periodic orbits in the parameter plane of the Lorenz-84 low-order atmospheric circulation model [1].These *tricorns* appear inside *tongue*-like structures that born through simple Shi'lnikov bifurcations [2,3].As we increase the coefficient b of the model, these *tongues* invade the background zone where a fixed point attractor "lives" [Fig. 1(a)] and extends over the zone of stable orbits of period-1 [Fig. 1(b)]. We give a detailed description of the birth of these *tricorn*-like structures [Fig. 1(a)-(d)], as a consequence of a foliation of the parameter space [Fig. 1(a)] that provokes a rich dynamics of overlap of parameter planes that produces an abundant multistability (see basins of attractors at Fig. 1(e)). As the third parameter b increases the *tricorn* exhibits a codimension-3 phenomenon rotating in clockwise and anti-clockwise directions in the plane of the symmetric F vs. the asymmetric G thermal forcing parameters (see Fig. 1(c) and (d)).
Tricorns were reported in cubic maps by J. Milnor [4].and recently in a semiconductor laser model [5],but as far as we know it is the first time that they appear in the Lorenz-84 model. The numerical evidence presented here, confirm the isomorphism between bifurcations in fluids and lasers formally demonstrated by Haken [6],and also motivates the study of mathematical conditions for their genesis [2].Our results also open new possibilities to assess experimentally the onset of intransivity and the significance of several different climate scenarios due to the birth of these structures.

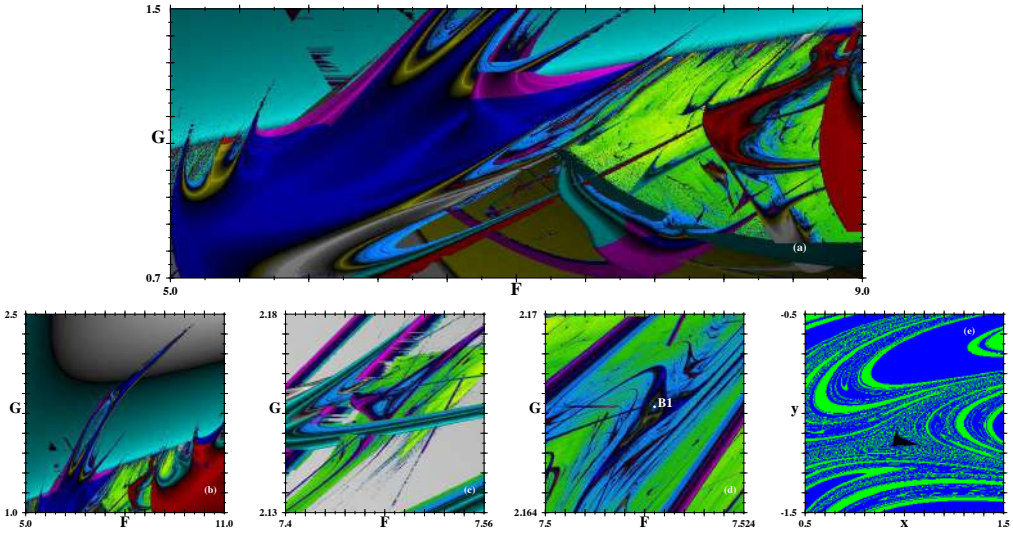

Figure 1: *First row: Global view of (F, G)-plane showing the invasion of tongue-like structures. Second row: process of formation and rotation of a tricorn-like structure. Basins showing the coexistence of periodic and chaotic attractors at point $B1$ (left).*

The authors acknowledges the use of the NEITOR-UMSS cluster located in Cochabamba, Bolivia and the "Núcleo de Investigación No.2, Sistemas Complejos en Ciencia e Ingeniería, UCN-VRIDT 042/2020" for the scientific support.

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Simulation of experimental front microrheology using a Non-Linear Klein Gordon equation with ϕ^4 potential.

E.S Figueroa[1*], M. García-Ñustes[1,†], C.Trejo-Soto[1]
[1] *Instituto de Física, Pontificia Universidad Católica de Valparaíso, Casilla 4059, Valparaíso.*
[*] elram.figueroa@pucv.cl, † monica.garcia@pucv.cl

Abstrac
In this research, we propose a numeric model to simulate the dynamics of a microrheology front in the presence of a spatial imperfection. Using a ϕ^4 model with a localized force $F(x, y)$, we model a fluid-air interface moving inside a microchannel with a localized roughness at the wall. Numerical simulation shows that the front begins to oscillate once it reaches the imperfection and the oscillation amplitude decays with time. The oscillation regime depends on the speed with which the front impact the defect.

Method
The system is described by the equation

$$\partial_{tt}u - \partial_{xx}u + \gamma\partial_t u(x,t) + \frac{1}{2}(u - u^3) = f_0 + F(x, y),$$

with f_0 and $F(x, y)$ a constant and a localized force respectively. The form of $F(x, y)$ is

$$F(x, y) = \frac{1}{2}(4B^2 - 1)\frac{\sinh Bx}{\cosh^3 Bx}e^{-\frac{y^2}{\sigma^2}}$$

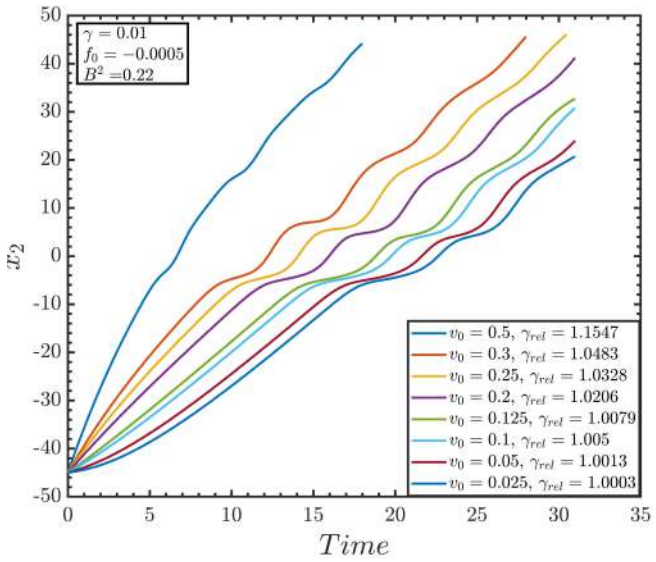

Figure 1: *Movement of the front for different initial velocities.*

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Confinement of chaotic solitons by the interplay of periodic spatio-temporal fields

Sorge Oporto-Almaraz*, Gerardo Meyer-Forgues, Camilo Castro, Deterlino Urzagasti†

Facultad de Ciencias Puras y Naturales, Universidad Mayor de San Andrés, Campus Universitario, Calle 27 Cota-Cota, La Paz, Bolivia

*sorgeoporto@gmail.com, †deterlino@yahoo.com

Introduction

Solitons are typical solutions of nonlinear Schrödinger equation (NLSE); this can be modified by the inclusion of several terms in order to account for different physical phenomena regarding a particular physical system under consideration. We use the model described by a general cubic order forced and parametrically driven and damped nonlinear Schrödinger equation (cubic-FPDDNLSE) in 1+1 dimensions for a complex amplitude function of the space-time coordinates, $A = A(x,t)$:

$$i\partial_t A - \sigma\partial_x^2 A + \gamma|A|^2 A = (\nu - i\mu)A + i\eta A^* - AN_0\sin^2(Kx);$$

besides the standard form of the NLSE (if $\sigma = 1$ and the right side term equals zero) we have a parametrically-driven and damped term (in blue) and a spatially periodic term (in violet) due to a external potential with amplitude N_0 and frequency K; η and ν are, respectively, the intensity and detuning frequency parameters of driving field, while μ is a damped parameter due to energy dissipation. The aim of this research is to study the triggering of localized and confined chaos in cubic-FPDDNLSE by the joint action of periodic temporal and spatial external fields.

The problem and results

Numerical simulations in 1 + 1 dimensions, Lagrange and Hamiltonian formulations for continuous fields, and global analyze for an hyperbolic-secant function A are used to study chaotic solutions [1, 2]. For $N_0 > 0$ the system triggers the chaotic and confined behavior of the soliton that we observe in the simulations as long as $\eta > 0$. In this sense, we could talk about a sort of interplay of the external both temporal driving field and spatial periodic potential: The former supports the existence of the soliton against the damping, while the latter originates its confinement and chaotic behavior, leading to trapped chaotic space-localized structures. Fig. **??** shows the evolution in space-time map of the amplitude $|A|$ when the spatial field is turn off and then it is turn on. The confinement effects in 1+2 and 1+3 dimensions are briefly explored considering the actions of a 2D and a 3D hexagonal spatial fields.

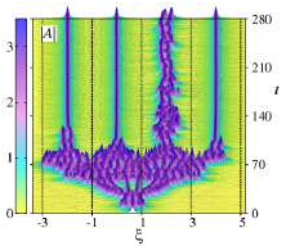

Figure 1: *Spatio-temporal map of the amplitude $|A|$ corresponding to the parameters $\nu = -3$, $\sigma = 1$, $\gamma = -1$, $K = 0.2$, $\nu = 2.844$, and $N_0 = 1.767$ where the external spatial field is turned on only from $t = 70$; the dashed lines represent the maxima of this filed and $\xi = 2Kx/\pi$*

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On the understanding of diffusion equation from maximum caliber principle

Diego González Díaz[1,2*]

[1] *Departamento de Física, Universidad Católica del Norte, Casilla 1280, Antofagasta, Chile*
[2] *Banco Itaú-Corpbanca, Casilla 80-D, Santiago, Chile*

[*] diego.gonzalez@ucn.com

Introduction

In the framework of statistical physics, in particular kinetic theory [1,2], the temporal evolution of a probability density for a Brownian motion is described by a partial differential equation called the diffusion equation,

$$\frac{\partial}{\partial t}P \; + \; D \; \frac{\partial^2}{\partial x^2}P = 0, \tag{1}$$

In this work it is shown how when restricting the classical action of a free particle, the time-dependent probability function will be governed by the diffusion equation.
This derivation can be used as a framework to simulate classical dynamical systems by using statistical techniques such as Monte Carlo Metropolis[3].

The problem and results

To understand the origin of the diffusion equation from maximum caliber is necessary to use time-dependent constraints that involves this principle and understand the way for obtaining partial differential equations which involves the use of the continuity equation[4],

$$\frac{\partial}{\partial t}P(x|t,I) + \frac{\partial}{\partial x}\Big(\; P(x|t,I) \; v(x,t)\Big) = 0, \tag{2}$$

and to achieve it, is necessary to obtain the form of the flow speed probability $v(y,t)$,

$$v(x,t) \quad = \quad \int d\tilde{x} \;\; P[\tilde{x}|I,x,t] \; \dot{\tilde{x}} \; . \tag{3}$$

To solve the form of $v(x,t)$, it is necessary to find the probability functional, which is obtained through the principle of maximum caliber when free particle action is constrained

$$P[\tilde{x}|I] = \frac{1}{Z(\lambda)} \;\; \exp\Big(- \lambda \int_{t_0}^{t_f} \; dt \; \frac{m(t)}{2} \; \dot{\tilde{x}}^2\Big), \tag{4}$$

Acknowledgment

DG thanks Professors Sergio Davis and Sergio Curilef for their constant support in the development and understanding of the principle of maximum caliber and its relationship with non-equilibrium statistical physics.

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Fine-tuning and SOqC in a neuronal network model

Gustavo Menesse[1,*], Boris Marin[2,†] and Osame Kinouchi[1,§]

[1] *Departamento de Física, FFCLRP, Universidade de São Paulo, Ribeirao Preto, SP, 14040-901, Brazil*
[2] *Centro de Matemática, Computao e Cognicao, Universidade Federal do ABC, SP, 09606-070, Brazil*

[*] menesse.gem@usp.br, [†] boris.marin@ufabc.edu.br, [§] okinouchi@gmail.com

Introduction

Self-organized criticality is achieved if a critical point is an attractor of a given dynamical system [1], provided that criticality is robust to perturbations and therefore independent of particular fine-tuned details in the dynamics. However, most well-known SOC models are conservative, as in the case of Abelian sandpiles [2] where the dissipation parameter in the transmission of grains must be zero. Hence, these models are actually subject to fine-tuning, in the form of a conservation law. Additionally, in any SOC model phase transitions, and therefore criticality, exist only for zero or very small effective external field [3], which can also be seen as a form fine-tuning. In models for dissipative systems such as earthquakes, forest fires or neural networks, only self-organized quasi-criticality (SOqC) holds. In this case, the system performs stochastic oscillations around the critical point. Several of such models include continuous drive and dissipation, whose dynamics can be viewed as homeostatic mechanisms which tune the system toward the critical region. In SOqC, there is no fine-tuning associated with a conservation law. However, homeostatic mechanisms for SOqC introduce tuning requirements in their parameters [4]. Here, we will discuss this issue in the context of a simple homeostatic mechanism for SOqC in a neuronal network, which holds even in the presence of external fields. This work results specially interesting in the context of critical brain hypothesis.

Acknowledgments

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"Shrimps" annihilation leading to ring structures of regular behavior

Gonzalo Marcelo Ramírez-Ávila[1*], Juergen Kurths[2,†], Jason A.C. Gallas[3]

[1] *Instituto de Investigaciones Físicas. Universidad Mayor de San Andrés. La Paz, Bolivia*
[2] *Potsdam Institute for Climate Impact Research. P.O. Box 601203 14412 Potsdam, Germany*
[3] *Max Planck Institute for the Physics of Complex Systems. Dresden, Germany*

[*] mravila@fiumsa.edu.bo

Introduction

we present a new sequence of structures in the parameter plane denoting regular behavior of a dynamical system. The sequence mentioned above, shows that contrarily to a tendency of an accumulation point, The tendency is towards a ring structure as a result of a process in which, coalescent shrimps exhibit a kind of annihilation of two "shrimps". We investigate two models of very different characteristics, one describing a vertical-cavity surface-emitting laser (VCSEL) subject to orthogonal optical injection, and the other a population model of cancerous, healthy, and effector cells when ionizing radiation is applied over the ensemble of cells; a typical situation occurring in a radiotherapy treatment. Despite the huge differences between the dynamical systems associated to the latter models, both show this phenomenon. Additionally, we detected the same regular structures in the Hartley's circuit. Thus, the ubiquity of these structures is remarkable [1].

The problem and results

Using the dynamic equations of each system, we determined the regions in which appear the annihilation described above. We use both periodicities and largest Lyapunov exponents for the dynamical description of the studied systems. As a glimpse, we show in Fig. 1, the sequence giving rise to a regular ring structure as a result of the "shrimps" annihilation for the VCSEL model described in [2].

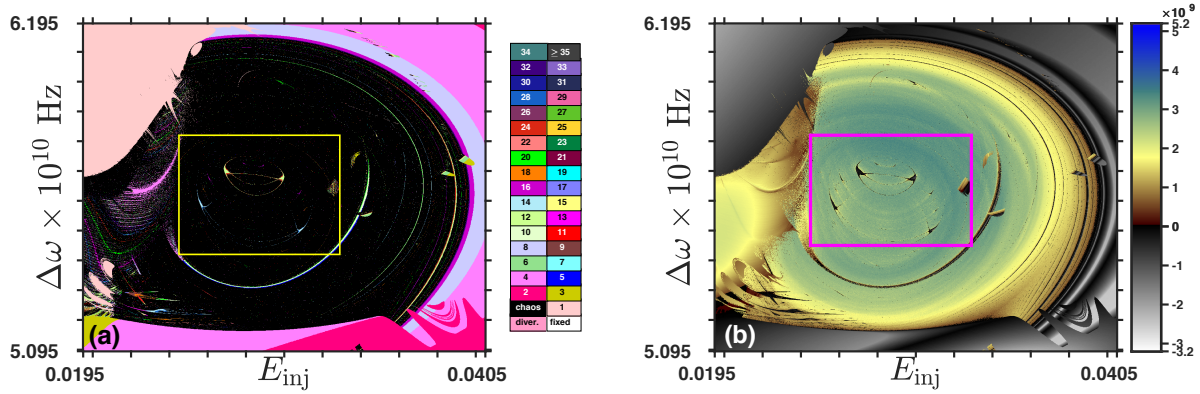

Figura 1: *Parameter plane $\Delta\omega$ vs. E_{inj} for the variable $E_{y\|}$ of the VCSEL model given by Eq. (1) in terms of (a) periodicities (number of isospikes), and (b) Largest Lyapunov exponents. Note that both diagrams permit to identify the regions of regular and chaotic behavior.*

Acknowledgment

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Geodynamic study of the dynamical instability of a low dimensional system of coupled anharmonic oscillators

Castillo. M. S.[1,*], Velázquez. L.[1,†], Calderón. F.[1,§]

[1] *Departamento de Física, Universidad Católica del Norte, Av. Angamos 0610, Antofagasta, Chile.*
[*] msc796@alumnos.ucn.cl, [†] lvelazquez@ucn.cl, [§] fcalderon@ucn.cl

Introduction

The proposed work deals with the dynamical instability of a system of coupled anharmonic oscillators of the type β-FPU Hamiltonian [1]:

$$H = \sum_{i=1}^{N} ({p_i}^2 + q_i^2) + \frac{1}{4}\beta(q_{i+1} - q_i)^4 \tag{1}$$

with periodic condition $q_{N+1} = q_1$ considering the geodynamic formalism [2]. According to this approach, once an appropriate metric g_{ij} is defined, the configuration space M of a system with Lagrangian coordinates $(q_1, q_2, ..., q_N)$ becomes a Rimmanian manifold. In such space, given a set of initial conditions, the natural motions of the system are geodesic flows $x^\mu(s, \tau)$ on M. This allows us to study the stability of the system by means of the Jacobi-Levi-Civita equation for geodesic spread:

$$\frac{\nabla^2 \xi^i}{ds^2} + R^i_{jkl}\frac{dq^j}{ds}\xi^k\frac{dq^l}{ds} = 0. \tag{2}$$

Here, R^i_{jkl} are the components of the curvature tensor, while $\xi^\mu(s, \tau)$ is the geodesic separation vector from a reference geodesic $\tau = 0$, given by

$$\xi^\mu(s, \tau) = \left[\frac{\partial x^\mu(s, \tau)}{\partial \tau}\right]_{\tau=0}. \tag{3}$$

As discussed elsewhere, the chaotic instability regimes are found by (i) the scatter of $x^\mu(s, \tau)$ due to negative curvature (either Scalar or Ricci curvature) on M, and (ii) by oscillations of the Ricci curvature along the geodesics so that parametric resonance makes them unstable also in regions of positive curvature [3]. Our interest is focused on the possible extension within this geometry formalism of the notion of Komolgorov-Sinai entropy and related results like Pesin's theorem concerning the sum of positive Lyapunov exponents [4]. For the same of convenience, we shall restrict here to a preliminary study of a low dimensional system (1) with $N = 3$, which is sufficient for our purposes.

Acknowledgments

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Intransitivity in the Lorenz-84 low-order atmospheric circulation model due to simple Shil'nikov bifurcations

Patricio Alvear Opazo[1*], C. Abraham T. Ch´avez(a)[2,†],Sergio Curilef[2,◇]
[1] *Universidad Católica del Norte, Av. Angamos 0610, Antofagasta, Chile*
[2] *Departamento de Ingenierías y Ciencias Exactas, Universidad Católica "San Pablo"-Tarija, Bolivia*
[3] *Facultad de Ciencias y Tecnología, Universidad Mayor de San Simón, Cochabamba, Bolivia*
[*] patricio.alvear@alumnos.ucn.cl, †ctorrico@ucb.edu.bo [◇]scurilef@ucn.cl

In this work, we report phase diagrams detailing the intransitivity as observed in the climate scenarios supported by a prototype atmospheric general circulation model, namely, the Lorenz-84 low-order model [1]. The first mechanism of multistability is the born of tongue-like structures due to simple Shil'nikov bifurcations [2], that invade the background region of a fixed point, FP-attractor. As generally believed, in the Lorenz-84 model the saddle-node bifurcation, SN-curve in Fig. 1(b), always takes place on a periodic orbit [3]. (This is also known as Adler's locking mechanism.) But, it has been recently shown that this does not happen along the whole of the curve SN [3]. As is also shown in Fig. 1(b) there are special points A1 and A2 where a homoclinic bifurcation curve touches SN (codimensiontwo saddle-node homoclinic points), giving rise to the formation of what has been called homoclinic tooth [3], inside which the homoclinic orbit bifurcates into an attracting periodic orbit that "lives" inside the tongue-like structure [Fig. 1(b)].
Bifurcation analysis of the parameter (F,G)-plane, through Lyapunov exponents scans [Figs. 1(a)-(b)] and the computation of basins [Fig. 1(c)], allows the identification of a remarkably wide parameter region where up to four climates coexist simultaneously [see a projection of four orbits on the (x,z)-plane in Fig. 1(d)]. As the coefficcient b of the Lorenz-84 model is increased, successive new tongue-like structures born along de SN bifurcation curve and invade the fixed point region [Fig. 1(a)], creating a rich scenario of multistability. Thus, in addition to the familiar sensitive dependence on initial conditions, the final climate (attractor) may depend crucially on the subtle and minute tuning of parameters [4]. This strong parameter sensitivity makes the Lorenz-84 model a promising candidate of testing ground to validate techniques of assessing the sensitivity of low-order models to perturbations of parameters.

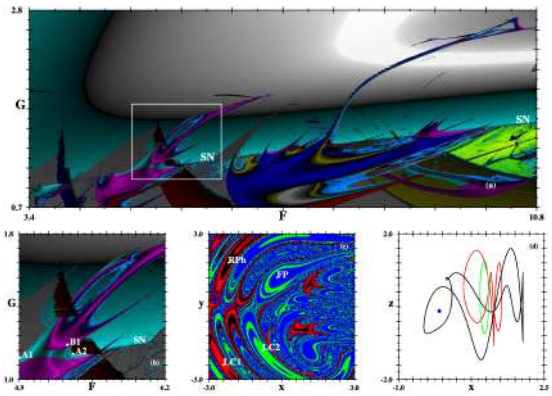

Figura 1: *First row: Global view of (F,G)-plane showing the birth of tongue-like structures along the saddle-node, SN bifurcation curve. Second row: (b) Enlargement of the tongue-like structure inside the box of the global view above. (c) basins of attraction at point B1 (left). (d) Projection of the orbits of four coexisting attractors at point B1 (left), on the phase plane-(x,z).*

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Drift instabilities in Faraday waves on a fluid experiment

Juan F. Marín[1,*], Rafael Riveros Ávila[2,†], Saliya Coulibaly[3], Mónica A. García-Ñustes[2], Majid Taki[3]

[1] *Departamento de Física, Universidad de Santiago de Chile, Av. Ecuador 3493, Estación Central, Santiago, Chile*
[2] *Instituto de Física, Pontificia Universidad Católica de Valparaíso, Casilla 4059, Chile*
[3] *Université de Lille, CNRS, UMR 8523, Laboratoire de Physique des Lasers Atomes et Molécules, F-59000 Lille, France*

[*] juan.marin.m@usach.cl, [†] rafael.riveros.a@mail.pucv.cl

Introduction

A well-known model for the study of pattern formation and its dynamics is the parametrically driven nonlinear Schrodinger (pndlS) equation. In particular, this equation can describe the envelope of Faraday waves at a certain region of parameters[1]. The equation reads as

$$\partial_t \psi = -(\mu + i\nu)\psi - i\alpha\partial_{xx}\psi - i|\psi|^2\psi + \gamma\psi^* \tag{1}$$

which is written in dimensionless form. For the pattern formation, the drive injection must overcome the energy dissipation, which is reflected on the amplitude equation as the condition $\gamma > \mu$.
Recently it has been pointed out that under spatially heterogeneous forcing profiles, in particular over Gaussian pro

files $\gamma(x)$, induce pattern envelopes given by the Gauss-Hermite polynomials[2]. Experiments on pattern formation at the water surface under heterogeneous forcing has shown correspondence with theoretical predictions[2,3].

Experimental Results

This talk will give an insight into the appearance of different phenomena related to the increase of the Gaussian injection amplitude of an experimental water system. In this region, non-linear effects get more relevant in the system description. Two clear regions are identified at different thresholds. The first one evidences a unidirectional drift, characterized by an increasing velocity related to amplitude growth. These results were previously predicted theoretically and numerically, and their behavior is well explained by the amplitude equation describing the spatial envelope of pndlS.

Other phenomena will be also discussed, the experiment presents other dynamics that seems to not be captured by the pndlS equation and deserve a future characterization and future theoretical description.

Experimental Setup and Data Management

The experimental setup consist of a rectangular water channel 15 mm long, 490 mm wide and 100 mm deep, whose bottom has a central soft region 240 mm wide (Shore hardness 00). The assembly rests over a system of 13 pistons evenly spaced ($\Delta x = 16$ mm), each of them constrained to vertical motion by two fixed axial bearings. The pistons are coupled to a brushless motor with feedback (Model No. BLM-N23-50-1000-B) controlled by computer to set the system's forcing frequency and amplitude. A VEO-440L Phantom camera was used for data acquisition, the frames where analysed with standard contour detection techniques.

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Dynamical analysis of massless charged particles

Zui Oporto[1,2*], Gonzalo Marcelo Ramírez-Ávila[1,†]

[1] *Instituto de Investigaciones Físicas, Universidad Mayor de San Andrés, La Paz, Bolivia.*
[2] *Planetario "Max Schreier", Universidad Mayor de San Andrés, La Paz, Bolivia*

[*] zoporto@fiumsa.edu.bo, [†] mravila@fiumsa.edu.bo

Introduction

An extensive list of works is dedicated to understanding the dynamics of point-charged particles (see [1] for a recent review). Massless charged particles represent a special case of study because of the absence of experimental evidence in favor of their existence as a fundamental particles, but at the same time because of its realization as (quasi-)particles in other physical scenarios such as the physics of graphene, just to mention one example. In this work, we present a dynamical system that describes the behavior of charged particles, relativistic or not, of arbitrary mass, including massless ones. We will consider some examples to illustrate the special features of this dynamical system.

The problem and results

Relativistic theories exhibit reparametrization invariance; consequently, its dynamics will be constrained by an algebraic equation that restricts the space of admissible solutions. Thus, as a dynamical system problem, we must face the situation of solving a set of first order differential equations while dealing with an algebraic equation[1]. This work presents a scheme to handle this constrained dynamical system by including the constraint as a differential equation. The aspect mentioned above is due to enlarging the number of independent degrees of freedom. In addition, we introduce the velocity space–a compact sphere–to track the dynamics of the charged particles. Figure 1 represents the trajectory–both in the physical as well as in the velocity space–of a massless charged particle interacting with an extenal electromagnetic field, in fact, of a circular polarized electromagnetic wave traveling in the z-direction.

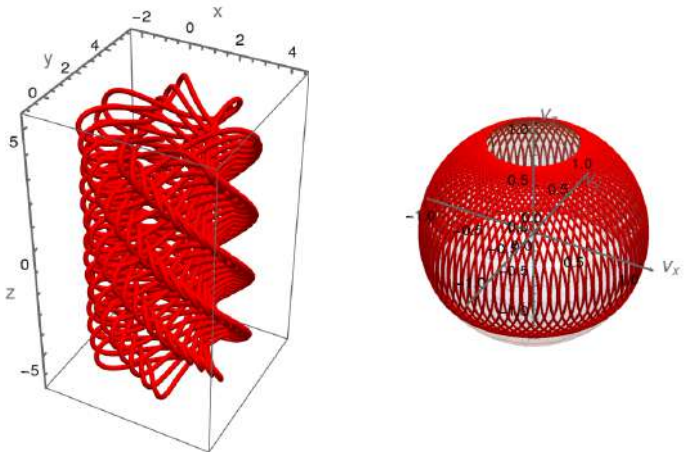

Figura 1: *(Left) Orbit in physcial space of a massless charged particle in the presence of a circularly polarized electromagnetic wave. (Right) The corresponding orbit in velocity space.*

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Multiplicative noise can induce a velocity change of propagating dissipative solitons

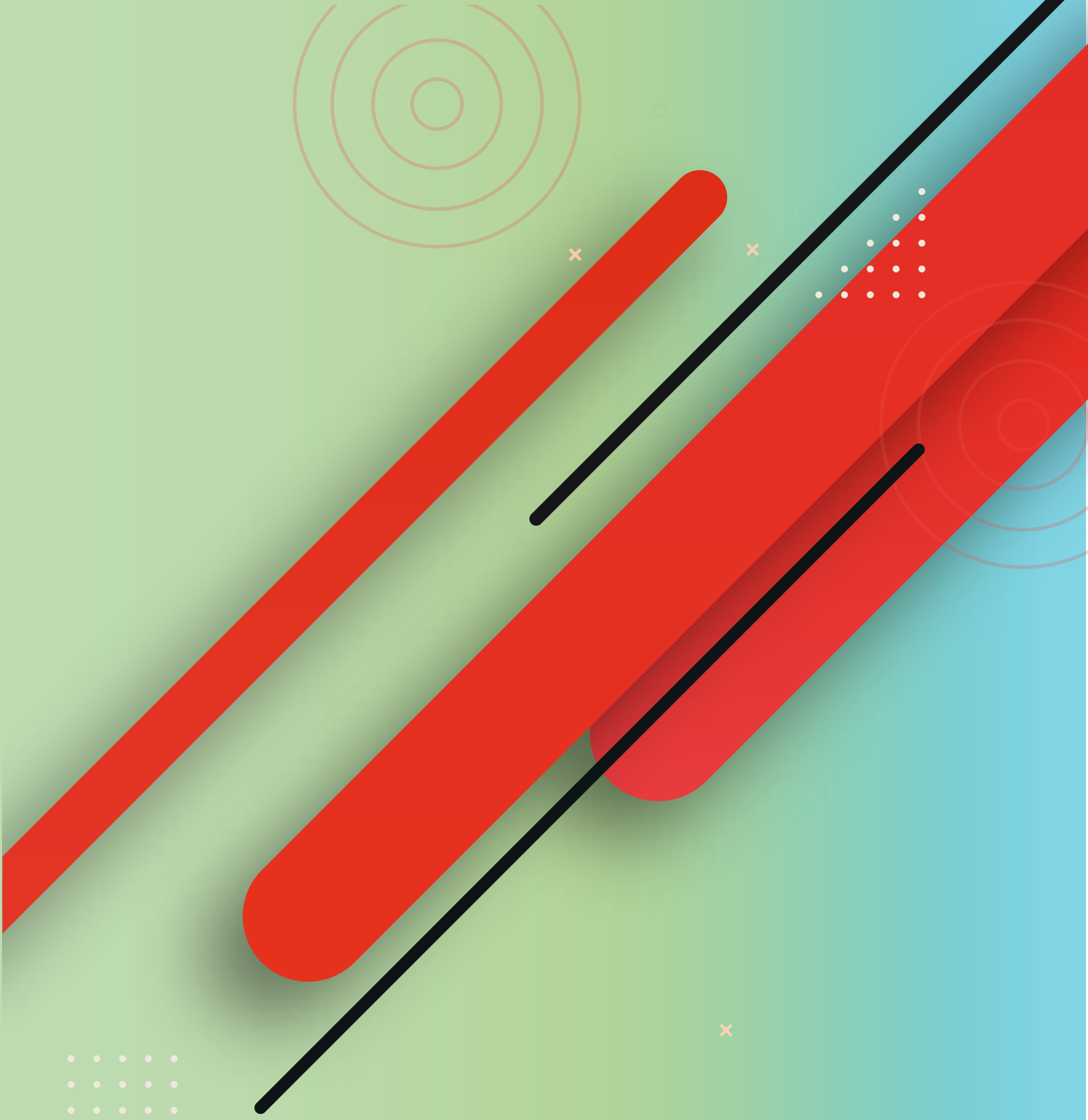
Orazio Descalzi[1*], Carlos Cartes[2], Helmut R. Brand[2]

[1] *Universidad de los Andes, Chile. Complex Systems Group. Facultad de Ingeniería y Ciencias Aplicadas.*
[2] *Department of Physics, University of Bayreuth, 95440 Bayreuth, Germany.*

[*] odescalzi@miuandes.cl

Introduction

We investigate the influence of spatially homogeneous multiplicative noise on propagating dissipative solitons (DSs) of the cubic complex Ginzburg-Landau equation stabilized by nonlinear gradient terms. Here we focus on the nonlinear gradient terms in particular on the influence of the Raman term and the delayed nonlinear gain. We demonstrate that a fairly small amount of multiplicative noise can lead to a change in the mean velocity for such systems. The effect can be traced back exclusively to the presence of the stabilizing nonlinear gradient terms. For a range of parameters we find a velocity change proportional to the noise intensity for the Raman term as well as for delayed nonlinear gain. We present a straightforward mean field analysis to capture this simple scaling law. At sufficiently high noise strength the nonlinear gradient stabilized (NLGS) DSs collapse.



Departamento de Física Universidad Católica del Norte /Av. Angamos 0610, Antofagasta / Pabellón J-2
Tel : +56-55-2355520 email: depto.fisica@ucn.cl